VATT WORKING PAPERS

3

# The optimal tax treatment of housing capital in the neoclassical growth model

Essi Eerola
Niku Määttänen

Valtion taloudellinen tutkimuskeskus
Government Institute for Economic Research
Helsinki 2009

Essi Eerola
Government Institute for Economic Research. essi.eerola@vatt.fi

Niku Määttänen
Helsinki School of Economics and CEBR. niku.maattanen@hse.fi

The authors thank Markus Haavio, Seppo Honkapohja, Karlo Kauko, Erkki Koskela, Henry Ohlsson, Mats Persson, Jouko Vilmunen and Matti Virén as well as seminar participants at IIES, IIPF, HECER, Bank of Finland, VATT, and Annual Meeting of the Finnish Economic Association for useful comments and suggestions. This research project was initiated during Eerola's visit at the Bank of Finland Research Department. She wishes to thank the Department for its hospitality during her visit. Both authors gratefully acknowledge financial support from the Yrjö Jahnsson Foundation.

ISBN 978-951-561-843-6 (nid.)
ISBN 978-951-561-844-3 (PDF)

ISSN 1798-0283 (nid.)
ISSN 1798-0291 (PDF)

Valtion taloudellinen tutkimuskeskus
Government Institute for Economic Research
Arkadiankatu 7, 00100 Helsinki, Finland

Oy Nord Print Ab
Helsinki, March 2009

# The optimal tax treatment of housing capital in the neoclassical growth model

Government Institute for Economic Research
VATT Working Papers 3/2009

Essi Eerola – Niku Määttänen

## Abstract

In a dynamic setting, housing capital is both an asset and a consumption good. But should it be taxed like other forms of consumption or like other forms of capital? We analyze this question by considering the taxation of housing capital in a version of the neoclassical growth model. We derive the optimal tax treatment of housing capital vis-à-vis the tax treatment of both business capital and other forms of consumption allowing for relatively general preferences. We show that for a class of utility functions that includes the standard Cobb-Douglas function, the second-best optimum can be achieved with a simple tax structure where housing construction is taxed at the same rate as non-housing consumption and the tax rate on the imputed rent equals the tax rate on the return to business capital in every period. We also show how the optimal tax structure depends on the elasticities of substitution between housing, non-housing consumption, and leisure. Our numerical analysis shows that the optimal tax burden on housing capital is indeed very sensitive to household preferences.

Key words: Optimal taxation, dynamic Ramsey taxation, housing taxation

JEL classification numbers: H21, E21

## Tiivistelmä

Omistusasunto on sekä varallisuuserä että kulutushyödyke. Mutta pitäisikö asumista verottaa kuten muuta kulutusta vai kuten muuta säästämistä? Tarkastelemme asuntovarallisuuden optimaalista verokohtelua dynaamisessa yleisen tasapainon mallissa. Johdamme asumisen optimaalisen verokohtelun suhteessa muun pääoman ja muun kulutuksen verotukseen. Näytämme, että tietyillä oletuksilla kotitalouksien preferensseistä, jotka kattavat mm. Cobb-Douglas hyötyfunktion, on optimaalista verottaa omistusasumisen laskennallista tuottoa samalla veroasteella kuin tuotannollista pääomaa, jos uudisrakentamista verotetaan samalla veroasteella kuin muuta kulutusta. Näytämme myös, miten tulos muuttuu, jos tarkastellaan yleisempää hyötyfunktiota. Tutkimuksen numeerisessa osassa osoitamme, että omistusasumisen optimaalinen verokohtelu on herkkä asumisen, muun kulutuksen ja vapaa-ajan välisille substituutiojoustoille.

Asiasanat: Optimiveroteoria, dynaaminen Ramsey-verotus, asuntovarallisuuden verotus

JEL-luokittelu: H21, E21

# 1 Introduction

The tax treatment of housing is an important fiscal question because housing wealth constitutes a large share of all household wealth. A common view in the public finance literature is that housing enjoys a tax favored status in most western economies, mainly because the return to owner housing, the imputed rent, usually goes untaxed while the return to business capital is taxed at a relatively high effective tax rate.[1] Related to this, several studies have assessed the welfare consequences of a tax reform that sets an equal tax burden on housing and business capital. Using quantitative dynamic general equilibrium models, Gahvari (1985), Skinner (1996), and Gervais (2002), among others, have shown that such a reform would lead to substantial efficiency gains.[2]

While these studies show that the current tax status of housing is likely to be highly distortionary, they do not aim to determine what the *optimal* tax treatment of housing is and how it depends on the overall tax system. We want to emphasize two reasons why it may not always be appropriate to call for tax reforms that equalize the effective tax rates on housing and business capital. Both of them are related to the consumption aspect of housing. First, it seems likely that the tax treatment of housing should depend on household preferences. For instance, we would like to know whether the optimal tax treatment of housing is very sensitive to the elasticity of substitution between housing and leisure.

Second, it seems clear that the tax rate on non-housing consumption should matter for the optimal tax treatment of housing. Consumption taxes are low in the US but an important source of tax revenue in many European countries. In the European Union, the standard rate of the value added tax (VAT) ranges from 15% in Luxembourg and

[1] See Hendershott and White (2000) for an international comparison of housing's tax status.

[2] Other studies that also consider the efficiency and welfare effects of the tax favored status of housing include Gahvari (1984a), Slemrod (1982), Berkovec and Fullerton (1992), Hendershott and Won (1992), Poterba (1992), and Bye and Åvitsland (2003) and Eerola and Määttänen (2006). Turnovsky and Okuyama (1994) focus solely on capital accumulation. See also Englund (2003) for general discussion on housing taxation.

Cyprus to 25% in Denmark and Sweden. At the same time, the relative tax treatment of housing vis-à-vis non-housing consumption within the VAT system varies a lot across the European countries. Some countries apply the standard VAT rate to the construction of residential housing, whereas others apply a reduced rate and some (United Kingdom) exempt the construction of residential housing altogether from VAT.[3]

In this paper, we try to understand these issues by analyzing the optimal tax treatment of housing capital vis-a-vis both business capital and non-housing consumption allowing for relatively general preferences. We employ a version of the neoclassical growth model with a representative household that derives utility from non-housing consumption, housing services, and leisure. The model captures the dual role of housing capital as both an asset and a form of consumption, the intertemporal savings-consumption decision, and the general equilibrium effects of capital taxation we are interested in. As far as we know, we are the first to consider the taxation of housing capital or durable goods as part of an optimal tax problem in a fully dynamic general equilibrium set-up.[4]

We follow the line of research represented by Judd (1985), Chamley (1986), Jones et al. (1997), Atkeson et al. (1999), and others. Formally, we analyze a Ramsey problem for a government that finances government expenditure by a set of flat rate taxes, including a tax on the imputed rental income from housing capital and a tax on housing construction. The government is assumed to be able to commit to future tax policies. The solution to the Ramsey problem is a tax reform which is optimal given the initial state of the economy, individual optimization, and the available tax instruments. In addition to introducing housing capital and a tax on it, we extend the most standard set-up with a consumption tax and a nested constant elasticity of substitution (CES) utility function.

[3]See Institute for Fiscal Studies (2004, chapter 5) for a discussion of housing taxation in the UK and elsewhere.

[4]Gahvari (1984b) studies the optimal taxation of housing capital in a partial equilibrium model. Cremer and Gahvari (1998) study the optimal taxation of housing in a static model with incomplete information, where the government may use differentiated housing taxes so as to separate between different consumer types.

We first derive analytical results. We show that for a class of utility functions that includes the standard Cobb-Douglas function, the second-best optimum can be achieved with a simple tax structure where construction is taxed at the same rate as non-housing consumption and the tax rate on the imputed rent equals the tax rate on the return to business capital in every period.[5] In addition, we show how the optimal tax structure depends on the elasticities of substitution between housing, non-housing consumption, and leisure. Depending on household preferences, it may be optimal to tax the imputed rent at a higher or lower rate than the return to business capital even when construction is taxed at the same rate as non-housing consumption.

In our numerical analysis, we first display the dynamics of optimal tax reforms. We then show that the optimal tax burden on housing capital is indeed very sensitive to the intratemporal elasticities of substitution between housing consumption, non-housing consumption, and leisure. Finally, we compute the welfare gains of optimal tax reforms. The most interesting exercise in this respect is to compare the welfare gains with and without the possibility to adjust the tax burden on housing. We find that the welfare gain of an optimal tax reform falls dramatically if the tax burden on housing cannot be adjusted.

We proceed as follows. In the next section we describe the economy. In section 3, we present our analytical results. In section 4, we present and discuss our numerical results. We conclude in section 5. In the Appendix, we derive our analytical results.

# 2 The model

We consider a deterministic model with an infinitely lived representative household that derives utility from non-housing consumption, housing services, and leisure. The production side consists of a representative firm that employs business capital and labor to produce output goods which can be transformed into housing capital, business capital,

[5] To be precise, this result holds for periods when exogenously imposed upper bounds for tax rates on business and housing capital are not binding.

and non-housing consumption. There is a government that finances public expenditures with flat-rate taxes.

We do not have residential land in the model. Hence, we focus on the tax treatment of housing capital, or residential structures, alone. One reason is that it seems obvious that if constructible land is in fixed supply, the government should try to effectively confiscate all land rents. In practice, this can be (partly) achieved with municipal monopoly power on decisions about land use. In this sense, adding land would not necessarily give interesting new insights.[6]

## 2.1 Firms

Every period $t$, a representative firm employs business capital, $k_t$, and labor, $n_t$, to produce output goods, $y_t$. The production function is

$$y_t = f(k_t, n_t). \tag{1}$$

We assume that the production function exhibits constant returns to scale. The firm's first-order conditions for profit maximization imply that the before-tax returns to business capital and labor are determined by their marginal productivities, that is,

$$r_t = f_{k_t} - \delta_k \tag{2}$$

$$w_t = f_{n_t}, \tag{3}$$

where $\delta_k$ is the depreciation rate of business capital.[7] The output good may be costlessly converted into non-housing consumption good, business capital, and housing capital.

[6] A potentially interesting way of introducing land into our analysis might involve modeling the development of 'raw land' and introducing a spatial aspect. However, these extensions would complicate our analysis substantially and would make it at least much more difficult to obtain analytical results about the optimal tax structure. See Aura and Davidoff (2006) for an interesting analysis about land taxation in a static spatial model and Davis and Heathcote (2005) for a calibrated business cycle model with housing and land.

[7] We denote $\frac{\partial}{\partial k_t} f(k_t, n_t) = f_{k_t}$ and similarly for other derivatives throughout the paper.

## 2.2 The household's problem

The household is endowed with one unit of time every period. The periodic utility function is $u(c, h, n)$, where $c$ is non-housing consumption, $h$ is the stock of housing capital, and $n$ labor. The utility function is assumed to be strictly increasing in non-housing consumption and housing capital and strictly decreasing in labor, strictly concave, and to satisfy the Inada conditions. The maximization problem of the household in period 1 is

$$\max \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, h_t, n_t) \tag{4}$$

subject to the intertemporal budget constraint

$$\sum_{t=1}^{\infty} p_t[(1+\tau_t^c)\, c_t + k_{t+1} + \left(1+\tau_t^i\right)(h_{t+1} - (1-\delta_h)\, h_t) - R_t k_t - (1-\tau_t^n) n_t w_t + \tau_t^h r_t^h h_t] \leq 0 \tag{5}$$

where

$$R_t = 1 + (1-\tau_t^k) r_t. \tag{6}$$

and $r^h$ denotes the imputed rent (defined below). In the budget constraint, $\tau^c$, $\tau^i$, $\tau^k$, $\tau^h$, and $\tau^n$ are the tax rates on non-housing consumption, housing construction, the return to business capital, the imputed rental income, and labor, respectively, and $p_t$ is the before-tax price of period $t$ non-housing consumption in terms of period 1 non-housing consumption. Parameter $\delta_h$ is the depreciation rate of housing capital. The tax on construction, $\tau^i$, increases the relative cost of existing housing capital with respect to non-housing consumption and business capital.

We define the imputed rent as the rental rate of housing, net of deprecation, that would prevail if rental markets existed in this economy. This hypotethical rental rate, let us denote it by *rent*, is determined by an arbitrage condition. Investing one unit of output good in period $t-1$ in business capital returns $1 + (1-\tau_t^k)r_t$ units in period $t$. One unit of output good buys $1/(1+\tau^i)$ units of housing capital. We assume that $\tau^i$ is fixed (we will discuss this assumption later), that the return to rental housing is taxed at the same rate as the return to business capital, and that landlords can deduct housing capital depreciation before paying the capital income tax. Then, investing one unit of the output good in period $t-1$ in rental housing returns

$$\frac{1}{1+\tau^i}\left[rent_t - \tau_t^k\left(rent_t - \left(1+\tau^i\right)\delta_h\right) + \left(1+\tau^i\right)\left(1-\delta_h\right)\right]$$

in period $t$. The last term in this expression is the value of the remaining housing capital in period $t$. It is now straightforward to show that the rental rate that makes households indifferent between investing in business capital and rental housing is $rent_t = (1+\tau^i)(r_t + \delta_h)$. Hence, the imputed rent (rental rate net of depreciation) is

$$r_t^h = (1+\tau^i)r_t. \tag{7}$$

Note that this is also the capital cost of one unit of housing.[8]

It is important to understand, however, that since we allow the tax rate on the imputed rent to be time varying, the allocations associated with optimal tax reforms do not depend on how exactly we define the tax base in housing taxation. In particular, housing taxation could equivalently take the form of a property tax.

The first-order conditions characterizing individually optimal behavior may be written as

$$\begin{aligned} u_{c_t}(1-\tau_t^n)w_t + u_{n_t}(1+\tau_t^c) &= 0 \qquad (8)\\ \frac{u_{c_t}}{1+\tau_t^c} - \frac{\beta u_{c_{t+1}}}{1+\tau_{t+1}^c}R_{t+1} &= 0 \qquad (9)\\ \beta u_{h_{t+1}} - u_{c_t}\frac{1+\tau_t^i}{1+\tau_t^c} - \beta u_{c_{t+1}}\left(1-\delta_h\right)\frac{1+\tau_{t+1}^i}{1+\tau_{t+1}^c} + u_{c_{t+1}}\frac{\tau_{t+1}^h r_{t+1}^h}{1+\tau_{t+1}^c} &= 0. \qquad (10) \end{aligned}$$

## 2.3 Government

Public consumption in each period is denoted by $g$. We assume that the government budget need not be balanced on a period by period basis. Instead, the government faces

[8] It would be natural to let the household deduct mortgage interest payments from the (gross) imputed rent before taxing it. However, our representative household does not hold debt.

the following intertemporal budget constraint:

$$\sum_{t=1}^{\infty} p_t \left( \tau_t^c c_t + \tau_t^k r_t k_t + \tau_t^n n_t w_t + \tau_t^h r_t^h h_t + \tau_t^i \left( h_{t+1} - (1 - \delta_h) h_t \right) - g \right) \geq 0. \tag{11}$$

## 2.4 Equilibrium

For a given sequence of tax rates, a competitive equilibrium consists of individual policies and prices such that the individual policies solve the household's problem in (4) and (5), factor returns are given by equations in (2) and (3), the government budget constraint in (11) is satisfied, and the aggregate resource constraint

$$c_t + k_{t+1} + h_{t+1} + g = f(k_t, n_t) + (1 - \delta_k) k_t + (1 - \delta_h) h_t \tag{12}$$

is satisfied for all $t$.

## 2.5 Specification of the preferences

For most of our analytical results, and obviously for all numerical results, we need to specify the utility function. We consider the following nested constant elasticity of substitution (CES) utility function.

The first CES aggregator, $\widehat{c}$, is defined over housing capital and leisure:

$$\widehat{c} = \widehat{c}(h, 1-n) = \begin{cases} \left( \theta^h h^{\gamma^h} + (1 - \theta^h)(1-n)^{\gamma^h} \right)^{1/\gamma^h}, \text{ for } \gamma^h < 1, \, \gamma^h \neq 0 \\ h^{\theta^h} (1-n)^{1-\theta^h}, \text{ for } \gamma^h = 0, \end{cases}$$

where $\theta^h$ is the utility share of housing capital. The elasticity of substitution between housing capital and leisure is given by $\frac{1}{1-\gamma^h}$.

The second CES aggregator, $\widetilde{c}$, is defined over non-housing consumption and $\widehat{c}$:

$$\widetilde{c} = \widetilde{c}(c, \widehat{c}) = \begin{cases} \left( \theta^c c^{\gamma^c} + (1 - \theta^c) \, \widehat{c}^{\gamma^c} \right)^{1/\gamma^c}, \text{ for } \gamma^c < 1, \, \gamma^c \neq 0 \\ \left( c^{\theta^c} \widehat{c}^{1-\theta^c} \right), \text{ for } \gamma^c = 0, \end{cases}$$

where $\theta^c$ is the utility share of non-housing consumption. The elasticity of substitution between non-housing consumption and $\widehat{c}$ is given by $\frac{1}{1-\gamma^c}$.

Finally, the periodic utility function is:

$$u(c,h,n) \equiv \widetilde{u}(\widetilde{c}(c,\widehat{c}(h,1-n)) = \begin{cases} \frac{\widetilde{c}^{1-\sigma}}{1-\sigma}, \text{ for } \sigma > 0, \sigma \neq 1 \\ \log(\widetilde{c}), \text{ for } \sigma = 1, \end{cases}$$

where $\sigma$ is the inverse of the intertemporal elasticity of substitution.

With $\gamma^c = \gamma^h = 0$, this utility function boils down to the commonly used Cobb-Douglas specification:

$$u(c,h,n) = \frac{[c^{\theta^c} h^{(1-\theta^c)\theta^h} (1-n)^{(1-\theta^c)(1-\theta^h)}]^{1-\sigma}}{1-\sigma}$$

The case with $\sigma = 1$ and $\gamma^c = \gamma^h = 0$ is the logarithmic utility function:

$$u(c,h,n) = \theta^c \log c + (1-\theta^c)\theta^h \log h + (1-\theta^c)(1-\theta^h) \log(1-n). \tag{13}$$

These preferences are equivalent to the set-up in Greenwood and Hercowitz (1991) who define a utility function over market consumption and 'home production' and a home production function over 'home capital' and time not allocated to market work.[9]

# 3 Optimal taxation

Before proceeding with the analysis of second-best taxation, we should first note that the fact that the government can tax consumption means that we can reach the first-best allocation with a constant strictly positive tax on non-housing consumption and construction, a constant subsidy on labor income (i.e. $\tau^n < 0$), and a zero tax on the return to business capital and the imputed rent (provided that government expenditures are sufficiently small). This solution effectively relies on taxing initial assets alone. A practical problem with this tax system is that the subsidy to labor would give households

[9] A more general formulation would allow for allocating time to 'leisure', 'home production' and 'market production'. For studies using this approach, see e.g. Gomme et al. (2001), Baxter and Jerman (1999) and McGrattan et al. (1997). The two approaches result in the same allocations under a logarithmic specification. For more discussion on this issue, see Greenwood et al. (1995).

a strong incentive to misrepresent hours of work. See Coleman (2000) for a discussion of these issues and an analysis of the case where government chooses a sequence of tax rates on consumption, labor and (business) capital subject to the constraint that the labor tax cannot be negative. It is also clear that if both $\tau^i$ and $\tau^h$ can be freely chosen, one of them is redundant.

In order to avoid these complications and be able to focus on the optimal tax treatment of housing, we take the consumption tax system as given by fixing $\tau^c$ and $\tau^i$. This rules out the possibility of taxing only the initial assets by imposing a very high tax rate on all forms of consumption together with a subsidy on labor. In our numerical experiments, this also suffices to make the optimal labor income tax rate strictly positive.

From now on therefore, we denote $\tau_t^c = \tau^c$ and $\tau_t^i = \tau^i$ and state the objective of the government as maximizing household welfare by announcing in the beginning of period 1 a sequence of tax rates $\{\tau_t^n, \tau_{t+1}^k, \tau_{t+1}^h\}_{t=1}^{\infty}$. Note that we also assume that the government takes as given $\tau_1^k$ and $\tau_1^h$. Otherwise, the government could trivially reach the first-best allocation by simply confiscating part of the existing capital stocks. In addition, and in line with the dynamic Ramsey literature, we define upper bounds $\overline{\tau}^k$ and $\overline{\tau}^h$ that the tax rates on the return to business capital and imputed rental income may not exceed in any period $t \geq 2$.

Following the approach taken by Chamley (1986), Judd (1985), and others, we formulate the government's problem so that it directly chooses allocation $\{c_t, n_t, k_{t+1}, h_{t+1}\}_{t=1}^{\infty}$.[10] Before writing down the government objective, let us discuss the constraints to be imposed on the government's choices.

Rewriting the budget constraint of the household in (5) by using the first-order

[10]As shown in Lansing (1999) and discussed in Krusell (2002), in some cases, this approach leads to allocations that cannot be decentralised. This happens in the absence of anticipation effects, that is, when future tax rates do not affect the current decisions of the private sector. In our setting this kind of anticipation effects are always present even under logarithmic utility.

conditions of the household gives

$$\sum_{t=1}^{\infty} \beta^{t-1} \left( u_{c_t} c_t + u_{n_t} n_t + \beta u_{h_{t+1}} h_{t+1} \right) = u_{c_1} \eta A. \tag{14}$$

where $A = R_1 k_1 + (1+\tau^i)(1-\delta_h) h_1 - \tau_1^h r_1^h h_1$ and $\eta = \frac{1}{1+\tau^c}$. This is the so-called implementability constraint. It states that the allocation chosen by the government must be compatible with individual optimization.

The upper bounds on the two capital tax rates are imposed as follows: First, given (9), the requirement $\tau_t^k \leq \overline{\tau}^k$ means that the government is constrained to choose allocations that satisfy

$$u_{c_{t-1}} \geq \beta u_{c_t} \left( 1 + (1-\overline{\tau}^k) r_t \right).$$

Second, given (10), the requirement $\tau_t^h \leq \overline{\tau}^h$ means that the allocations chosen by the government must satisfy

$$\left(1+\tau^i\right) u_{c_{t-1}} \eta \geq \beta u_{h_t} + \beta u_{c_t} \eta \left( \left(1+\tau^i\right)(1-\delta_h) - \overline{\tau}^h r_t^h \right).$$

The Lagrangian for the government may be written as:

$$\begin{aligned}
\mathcal{L} \;=\; & \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, h_t, n_t) \\
& + \lambda \left[ \sum_{t=1}^{\infty} \beta^{t-1} \left( u_{c_t} c_t + u_{n_t} n_t + \beta u_{h_{t+1}} h_{t+1} \right) - u_{c_1} \eta A \right] \\
& + \sum_{t=1}^{\infty} \beta^{t-1} \mu_t \left[ f(k_t, n_t) + (1-\delta_k) k_t + (1-\delta_h) h_t - c_t - k_{t+1} - h_{t+1} - g \right] \\
& + \sum_{t=1}^{\infty} \beta^{t-1} \omega_t \left[ u_{c_t} - \beta u_{c_{t+1}} \left( 1 + (1-\overline{\tau}^k) r_{t+1} \right) \right] \\
& + \sum_{t=1}^{\infty} \beta^{t-1} \psi_t \left[ \left(1+\tau^i\right) u_{c_t} \eta - \beta u_{c_{t+1}} \eta \left( \left(1+\tau^i\right)(1-\delta_h) - \overline{\tau}^h r_{t+1}^h \right) - \beta u_{h_{t+1}} \right].
\end{aligned}$$

The first constraint is the implementability constraint. The second set of constraints contains an aggregate resource constraint for each period. The third and fourth sets of constraints are the restrictions on the tax rates.

For $t > 1$, the first-order conditions for $n_t$, $c_t$, $k_{t+1}$, and $h_{t+1}$ are:[11]

$$W_{n_t} + \mu_t f_{n_t} + B_{t-1} u_{cn_t} - \omega_{t-1}\left(1-\overline{\tau}^k\right) u_{c_t} f_{kn_t} - \psi_{t-1}\left[u_{hn_t} - \eta u_{c_t} \overline{\tau}^h \frac{\partial r_t^h}{\partial n_t}\right] = 0 \quad (15)$$

$$W_{c_t} - \mu_t + B_{t-1} u_{cc_t} - \psi_{t-1} u_{hc_t} = 0 \quad (16)$$

$$-\mu_t + \beta\mu_{t+1}\left(1 + r_{t+1}\right) - \omega_t\left(1-\overline{\tau}^k\right) u_{c_{t+1}} f_{kk_{t+1}} + \psi_t \eta u_{c_{t+1}} \overline{\tau}^h \frac{\partial r_{t+1}^h}{\partial k_{t+1}} = 0 \quad (17)$$

$$\beta W_{h_{t+1}} - \mu_t + \beta\mu_{t+1}\left(1-\delta_h\right) + \beta B_t u_{ch_{t+1}} - \psi_t \beta u_{hh_{t+1}} = 0 \quad (18)$$

where

$$\begin{aligned}
B_t &= \omega_{t+1} - \omega_t\left(1 + (1-\overline{\tau}^k) r_{t+1}\right) + \psi_{t+1}\left(1+\tau^i\right)\eta - \psi_t \eta\left(\left(1+\tau^i\right)\left(1-\delta_h\right) - \overline{\tau}^h r_{t+1}^h\right) \\
W_{n_t} &= u_{n_t} + \lambda(u_{hn_t} h_t + u_{cn_t} c_t + u_{nn_t} n_t + u_{n_t}) \\
W_{c_t} &= u_{c_t} + \lambda(u_{hc_t} h_t + u_{cc_t} c_t + u_{c_t} + u_{nc_t} n_t) \\
W_{h_t} &= u_{h_t} + \lambda(u_{hh_t} h_t + u_{h_t} + u_{ch_t} c_t + u_{nh_t} n_t).
\end{aligned}$$

We also have the following Kuhn-Tucker conditions for all $t \geq 1$:

$$\psi_t\left(u_{c_t}\left(1+\tau^i\right)\eta - \beta u_{c_{t+1}}\eta\left(\left(1+\tau^i\right)\left(1-\delta_h\right) - \overline{\tau}^h r_{t+1}^h\right) - \beta u_{h_{t+1}}\right) = 0,\ \psi_t \geq 0, \quad (19)$$

$$\text{and } u_{c_t}\left(1+\tau^i\right)\eta - \beta u_{c_{t+1}}\eta\left(\left(1+\tau^i\right)\left(1-\delta_h\right) - \overline{\tau}^h r_{t+1}^h\right) \geq \beta u_{h_{t+1}}.$$

$$\omega_t\left(u_{c_t} - \beta u_{c_{t+1}}\left(1 + (1-\overline{\tau}^k) r_{t+1}\right)\right) = 0,\ \omega_t \geq 0, \quad (20)$$

$$\text{and } u_{c_t} - \beta u_{c_{t+1}}\left(1 + (1-\overline{\tau}^k) r_{t+1}\right) \geq 0.$$

The optimality conditions (15)-(20), the aggregate resource constraint (12), and the implementability constraint (14) determine the allocation, $\{c_t, n_t, k_{t+1}, h_{t+1}\}_{t=1}^{\infty}$, as well as the multipliers $\lambda$ and $\{\mu_t, \omega_t, \psi_t\}_{t=1}^{\infty}$. After an optimal allocation has been found, prices, $\{r_{t,} w_t\}_{t=1}^{\infty}$, are determined from equations (2) and (3). Finally, the tax rates on labor, the return to business capital, and the imputed rental income are solved from equations (8), (9), and (10), respectively.

By combining the steady state versions of the government first-order condition (17) and the household first-order condition (9), it is straightforward to show that in the long

[11] The first-order conditions for $c_1$ and $n_1$ (not shown) look somewhat different because all decisions affect the righ-hand side of (14).

run the tax rate on the return to business capital should be zero. This is the standard Chamley-Judd result.

Our interest is in comparing the optimal tax treatment of housing capital to that of business capital and non-housing consumption not just in the steady state but also during the transition. In what follows, we do this with two analytical results. When deriving these results, we ignore the upper bounds on the capital tax rates, $\overline{\tau}^k$ and $\overline{\tau}^h$, which can be binding only for the first periods after the optimal tax reform is announced. Hence, strictly speaking, the following results hold starting from the first period when neither of these upper bounds is binding.

By using the government's first-order conditions (16)-(18) and assuming that $\omega = \psi = 0$, we obtain

$$\frac{W_{h_{t+1}}}{W_{c_{t+1}}} - (\delta_h + r_{t+1}) = 0. \tag{21}$$

By using household's first-order conditions (9) and (10), we obtain

$$\frac{u_{h_{t+1}}}{u_{c_{t+1}}} = \frac{(1+\tau^i)\left[R_{t+1} - (1-\delta_h)\right]}{1+\tau^c} + \frac{\tau^h_{t+1}(1+\tau^i)\, r_{t+1}}{1+\tau^c}. \tag{22}$$

Our strategy is to combine (21) and (22) in a manner that will allow us to determine the relationship of different tax rates. In order to do that, we consider the utility function defined in subsection 2.5. Details of the analysis are in the Appendix.

The first result applies to the situation where the elasticity of substitution between housing and leisure (determined by $\gamma^h$) is the same as the elasticity of substitution between non-housing consumption and the CES aggregator over housing and leisure (determined by $\gamma^c$).

**Result 1** *If $\gamma^c = \gamma^h$, the following holds in all periods:*

$$(1+\tau^c)(\delta_h + r_{t+1}) = \left(1+\tau^i\right)\left[\left(1-\tau^k_{t+1}\right) r_{t+1} + \delta_h\right] + \tau^h_{t+1}\left(1+\tau^i\right) r_{t+1}.$$

Result 1 shows that the optimal tax rate on the imputed rent is positively related to the optimal tax rate on business capital. It also shows that the optimal tax rate on the imputed rent depends on consumption taxation. From Result 1 it directly follows that

**Corollary 1** *If* $\gamma^c = \gamma^h$ *and* $\tau^c = \tau^i$, *then* $\tau_t^k = \tau_t^h$ *in all periods.*

In words, for a class of utility functions that includes Cobb-Douglas preferences, the second-best optimum can be achieved with a tax structure where construction is taxed at the same rate as non-housing consumption and the tax rate on the imputed rental income equals the tax rate on the return to business capital.

Corollary 2 shows how the tax rate on the imputed rental income should be adjusted, if construction is untaxed.

**Corollary 2** *If* $\gamma^c = \gamma^h$ *and* $\tau^i = 0$, *then*

$$\tau_t^h = \tau^c \left(1 + \frac{\delta_h}{r_t}\right) + \tau_t^k.$$

If construction is not taxed but non-housing consumption is taxed, the tax rate on the imputed rent should exceed both the tax rate on non-housing consumption and the tax rate on the return to business capital.

Finally, we consider more general preferences:

**Result 2** *Assume that* $\tau^c = \tau^i$.

*i) If* $\gamma^c \neq \gamma^h$, *then* $\tau_t^k \neq \tau_t^h$ *in all periods.*

*ii) If* $\sigma = 1, \gamma^h > \gamma^c$, *and* $\gamma^h > 0$, *then* $\tau_t^h < \tau_t^k$ *in all periods.*

*iii) If* $\sigma = 1, \gamma^h < \gamma^c$, *and* $\gamma^h \leq 0$ *but close enough to zero, then* $\tau_t^h > \tau_t^k$ *in all periods.*

Part i) shows that whenever $\gamma^c \neq \gamma^h$, the result that the tax rate on the imputed rental income should equal the tax rate on the return to business capital breaks down. Parts ii) and iii) show that the imputed rental income should be taxed at a lower (higher) rate than the return to business capital if $\gamma^h$ is larger (smaller) than $\gamma^c$, at least when $\sigma = 1$ and $\gamma^h$ is not too small.

To interpret this result, note first that the elasticity of substitution between housing and leisure increases with $\gamma^h$. If the elasticity of substitution between housing and leisure is high, a small increase in the tax burden on housing leads the household to demand a lot more leisure (which cannot be taxed). In such a situation, the tax burden on housing should be low. However, the relevant elasticity depends also on $\gamma^c$. For a fixed $\gamma^h$, a higher value of $\gamma^c$ means that the household is more willing to substitute both housing and leisure for non-housing consumption. In other words, if $\gamma^c$ is high, a small increase in the tax burden on housing leads the household to demand a lot less leisure. Hence, the tax burden on housing should decrease with $\gamma^h$ and increase with $\gamma^c$.

More generally, this reflects the well-known result that goods that are sufficiently strong substitutes for leisure should be taxed at a lower rate than other goods (see e.g. Christiansen, 1984). However, we have not seen this result been derived before in a fully dynamic set-up.

# 4 Numerical analysis

In our view, the most important questions that our analytical results leave open are the following: First, what are the transitional dynamics of the optimal tax rates? Second, how sensitive is the optimal long run tax rate on imputed rental income with respect to changes in the CES preference parameters? Third, what is the welfare cost of not optimizing housing taxation? In this section, we present numerical results to answer these questions.[12]

## 4.1 Calibration

We take the model period to be one year. We assume that the production function is Cobb-Douglas with capital share $\alpha$. Greenwood et al. (1995) have estimated the share of business capital in the production function when total capital stock is disaggregated into housing and business capital. Based on their estimate, we set $\alpha = 0.29$. We set the

[12] We find the solution to the Ramsey problem by solving the system of non-linear equations formed by the government first-order conditions, the implementability constraint, the aggregate resource constraints, and the Kuhn-Tucker constraints using the broydn's algorithm. The Matlab programs are available from the authors upon request.

depreciation rates of business and housing capital so as to get the same investment to capital stock ratios as in the 2006 National Income and Product Accounts (NIPA) for the US. This implies $\delta_k = 0.096$ and $\delta_h = 0.042$.[13]

In the baseline calibration, we set $\gamma^c = \gamma^h = 0$. This is consistent with the fact that housing expenditure share and average hours worked have been fairly constant over the last decades, at least in the US (Kydland, 1995). However, studies using micro data often find different results.[14] We will conduct a sensitivity analysis with respect to both of these intratemporal elasticity parameters. In addition, we set $\sigma = 1$ so that we have the logarithmic utility function in (13).

We want the initial tax system to reflect a typical European economy with a relatively high overall tax burden as well as relatively high consumption taxes. Based on Carey and Rabesona (2004), we set $\tau^c = 0.18$, $\tau_0^n = 0.39$ and $\tau_0^k = 0.44$.[15] We assume that construction is taxed at the same rate as consumption, so that $\tau^i = \tau^c$. We further set the initial tax rate on the imputed rent at zero, i.e. $\tau_0^h = 0$. We choose the preference parameters $\beta$, $\theta^c$, $\theta^h$ and public expenditures, $g$, so as to match the following targets: 1) Total capital to total output ratio $(k + h)/y = 3.0$, where $y = k^\alpha n^{1-\alpha} + r^h h$. 2) Housing capital to business capital ratio $h/k = 0.78$. 3) Labor supply $n = 0.33$. 4) The government budget is balanced and there is no government debt.

Targets 1 and 2 are based on NIPA.[16] The first is the ratio of fixed assets (private and government) to GDP and the second is the ratio of residential fixed assets to non-

[13] These depreciation rates equal the ratios of gross private domestic investment in non-residential and residential fixed assets to the stocks of private non-residential and residential fixed assets.

[14] We are not aware of a study that would have estimated the same preference structure that we have here with micro data. However, there are a number of studies that use micro data to estimate the elasticity of substitution between housing and non-housing consumption. These studies tend to find that elasticity to be quite small. See for instance Siegel (2005) and Flavin and Nagazawa (2008).

[15] These numbers are based on the estimates for the effective tax rates on consumption, labor and capital in EU-15 countries in 1990-2000 (table 7.2 in Carey and Rabesona, 2004). We have adjusted them by the average effect (table 7.15) of the non-taxation of the imputed rental income.

[16] The reason we take these targets from the US data is that for we could not find measures of fixed asset stocks for large European countries.

residential fixed assets. The third target means that the representative household uses one third of its time endowment working. The last target fixes the constant public consumption. The implied ratio of tax revenue to GDP is 0.36.

Table 1: Baseline calibration.

| | Parameter | Value | Definition |
|---|---|---|---|
| Preferences | $\beta$ | 0.968 | Discount factor |
| | $\theta^c$ | 0.296 | Consumption share parameter |
| | $\theta^h$ | 0.128 | Housing share parameter |
| | $\gamma^c$ | 0 | CES parameter |
| | $\gamma^h$ | 0 | CES parameter |
| | $\sigma$ | 1 | Intertemporal elasticity parameter |
| Technology | $\alpha$ | 0.290 | Business capital share |
| | $\delta_k$ | 0.096 | Depreciation rate of business capital |
| | $\delta_h$ | 0.042 | Depreciation rate of housing capital |
| Tax system | $\tau_0^n$ | 0.390 | Initial labor tax |
| | $\tau_0^h$ | 0 | Initial tax rate on the imputed rent |
| | $\tau^c$ | 0.180 | Consumption tax |
| | $\tau^i$ | 0.180 | Tax on housing construction |
| | $\tau_0^k$ | 0.440 | Initial tax rate on the return to business capital |
| | $g$ | 0.171 | Government expenditures |

## 4.2 Transitionary dynamics

In this subsection, we display the optimal dynamic tax reform. We also illustrate how it changes if the imputed rental income cannot be taxed. In that case, we add to the government's problem the constraint that $\tau^h$ must equal zero in every period.[17] Figure 1 shows the paths of the optimal tax rates in the baseline calibration. Period 0 corresponds to the initial steady state. The tax rate paths in the left hand side of the figure correspond to the case where the imputed rent can be taxed. Here we have set the upper bounds

[17] Formally, the constraint reads as $u_{c_{t-1}}\eta = \beta u_{c_t}\eta\,(1-\delta_h) + \beta u_{h_t}$.

for the two capital tax rates at 1, that is, $\overline{\tau}^k = \overline{\tau}^h = 1$.[18] The tax rates paths in the right hand side corresponds to the case where the imputed rent cannot be taxed.

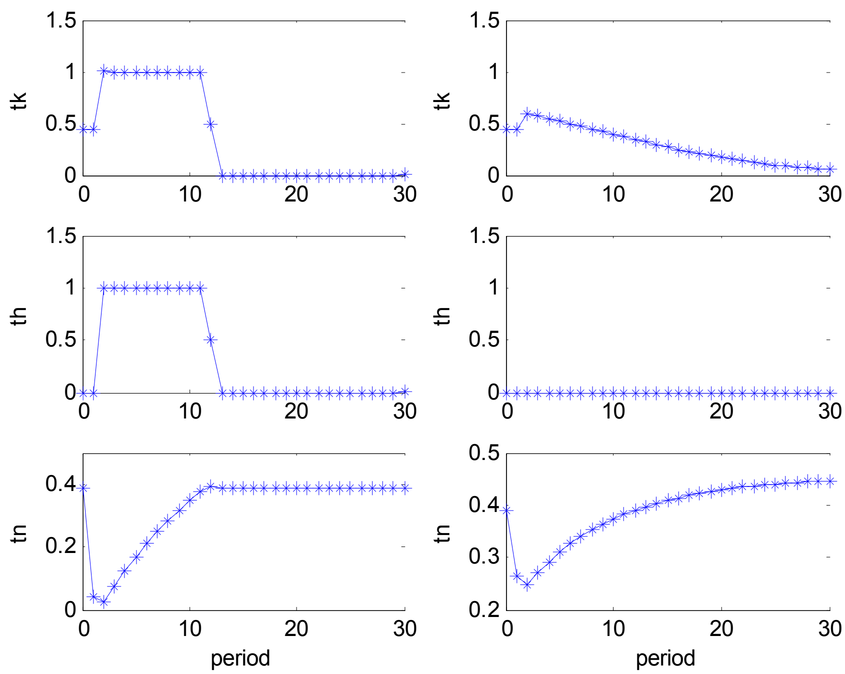


Figure 1: Optimal tax rates with (left) and without (right) the possibility to tax the imputed rent.

Consider first the case where the imputed rent can be taxed. The upper bounds for the tax rates on the imputed rent and the return to business capital are both binding until period 11. After that, the both tax rates converge to zero in one period. The labor tax falls close to zero during the first periods of the optimal tax reform.

[18]This is a natural upper bound for $\tau^k$ since the after-tax return to business capital would become negative with $\tau^k > 1$. In such a case, investors would refuse to hold any business capital. However, there is no such natural upper bound for $\tau^h$. This is because households should always be willing to hold some housing capital as long as the marginal utility of housing goes to infinity when housing goes to zero. In the absence of any restriction on the tax rate on the imputed rent, the optimal tax rate goes to about 2.1 in period 2 and then starts to decrease. The dynamics of $\tau^k$ do not change substantially from those in the left hand side of figure 1.

Interestingly, if the tax burden on housing cannot be adjusted, the dynamics of the optimal tax rate on the return to business capital are very different in two respects: First, the tax rate now starts to diminish from period 2 onwards and the upper bound $\tau^k \leq 1$ is never binding. Second, the tax rate on the return to business capital converges to zero very slowly. In fact, it appears to converge to zero only asymptotically.

In other words, if the tax burden on housing cannot be adjusted, the tax rate on the return to business capital does not feature the usual dynamics with very high tax rates in the first periods and a rapid convergence to the new steady state tax rate. Intuitively, taxing the initial business capital stock becomes undesirable if households can use housing capital as a tax free savings vehicle.

### 4.3 Elasticities and the optimal long-run tax system

We know from result 2 that the optimal tax rate on the imputed rent differs from the optimal tax rate on the return to business capital whenever the intratemporal elasticity parameters $\gamma^c$ and $\gamma^h$ are not equal. We now illustrate the quantitative importance of this result by reporting the optimal long run tax rate on the imputed rent with different values for these two parameters. We consider values of -1 and 1/3 which correspond to elasticities of substitution equal to 0.5 and 1.5, respectively. When changing these parameters, we recalibrate parameters $\beta$, $\theta^c$, $\theta^h$, and $g$, so as to match the same four targets that we match in the baseline calibration.[19] As before, the consumption tax, which applies also to construction, is fixed at 0.18. The optimal long run tax rate on the return to business capital is always zero.

In table 2, we display the optimal long run tax rates on the imputed rent and labor. In addition, we report the total tax burden on housing. We measure it as the relative increase in the user cost of housing caused by the taxation of the imputed rent and construction. The user cost is $(1+\tau^i)[(1+\tau^h)r+\delta_h]$. Without taxes, it would be $r+\delta_h$. In order to allow for an arbitrary large steady state tax rate on the imputed rent, we set

[19] In fact, $\beta$ remains constant when we vary the elasticity parameters.

here $\overline{\tau}^h = \infty$.

With a fixed tax on construction, the optimal tax rate on the imputed rent varies enormously with the elasticity parameters. In the table, the tax rate ranges from -0.98 to 2.63. The extreme cases are those where the difference between $\gamma^c$ and $\gamma^h$ is the largest. The variation in the overall tax burden is smaller: it ranges from -33% to 156%. Still, these results show that the optimal tax burden on housing is very sensitive to household preferences.

Table 2: Long run tax rates.

| Calibration | $\tau^h$ | $\tau^n$ | Total tax burden on housing (% of user cost) |
|---|---|---|---|
| $\gamma^c = 0, \gamma^h = -1$ | 1.69 | 0.25 | 107 % |
| $\gamma^c = 0, \gamma^h = 0$ | 0.00 | 0.38 | 18 % |
| $\gamma^c = 0, \gamma^h = 1/3$ | -0.52 | 0.42 | -9 % |
| $\gamma^c = -1, \gamma^h = 0$ | -0.81 | 0.46 | -24 % |
| $\gamma^c = 0, \gamma^h = 0$ | 0.00 | 0.37 | 18 % |
| $\gamma^c = 1/3, \gamma^h = 0$ | 0.82 | 0.29 | 61 % |
| $\gamma^c = -1, \gamma^h = 1/3$ | -0.98 | 0.47 | -33 % |
| $\gamma^c = 1/3, \gamma^h = -1$ | 2.63 | 0.17 | 156 % |

## 4.4 Welfare effects

Finally, we consider the welfare effects of optimal tax reforms. Our welfare measure is the 'equivalent consumption variation'. It tells how much consumption should be increased in the initial steady state (keeping housing and leisure fixed) so as to make the household indifferent between the status quo and the equilibrium associated with a tax reform. We compare the welfare gains of optimal tax reforms with and without the possibility to tax the imputed rent. The difference between these two welfare gains gives us a measure of the welfare cost associated with the inability to adjust the effective tax rate on housing capital from its initial level. When calculating the welfare effects, we take the transition into account.

With the baseline calibration, the welfare gain from an optimal dynamic tax reform is 5.4%. This welfare gain falls to 1.9% if the tax rate on the imputed rent is constrained to be zero. In other words, the welfare gain falls by about 65% if the imputed rent cannot be taxed. In this sense, the cost of not being able to tax the imputed rent is very large.

The optimal tax reform calls for very high capital tax rates during the first periods of an optimal dynamic tax reform as the government wishes to tax part of the initial capital stocks away. In the baseline calibration, we constrained the tax rates on the imputed rent and the return to business capital tax to be at most 1. As figure 1 shows, these constraints are binding during the first periods after the reform.

In order to assess the importance of the ability to impose very high tax rates during the first periods of the reform, we also computed optimal tax reforms with much lower upper bounds on the capital tax rates. Specifically, we assumed that the tax rate on the return to business capital and the tax rate on the imputed rent cannot exceed the initial tax rate on the return to business capital. That is $\overline{\tau}^k = \overline{\tau}^h = 0.44$. In this case, the welfare gain from the optimal tax reform is 3.5%. If the imputed rent cannot be taxed at all and $\overline{\tau}^k = 0.44$, the welfare gain is 1.8%.

Given that the initial tax burden on housing (with $\tau_0^h = 0$) is the same as the optimal tax burden in the new steady state, it is perhaps surprising that the welfare gain of the optimal tax reform falls so much if the tax burden on housing cannot be adjusted. The reason for the large reduction in the welfare gain is that taxing the initial business capital stock (through high tax rates on the return to business capital during the first periods of the reform) becomes difficult when households can use housing capital as a tax free savings vehicle.

# 5 Conclusions

We have considered the optimal tax treatment of housing capital within a version of the neoclassical growth model. The tax burden on housing should increase with both the taxation of non-housing consumption and the taxation of business capital. For a class of utility functions that includes the standard Cobb-Douglas function, the second-best optimum can be achieved by taxing construction at the same rate as non-housing consumption and taxing the imputed rent at the same rate as the return to business capital. However, the optimal tax burden on housing is very sensitive to the elasticities of substitution between housing consumption, non-housing consumption, and leisure. This means that the optimal tax rate on the imputed rent may depart substantially from the optimal tax rate on the return to business captital. We also found that the dynamics of optimal capital tax reforms are very different if the tax burden on housing cannot be adjusted.

# References

Atkeson, Andrew, V.V. Chari, and Patrick J. Kehoe (1999), Taxing Capital Income: A Bad Idea, *Federal Reserve Bank of Minneapolis Review* 23(3), 3-17.

Aura, Saku and Thomas Davidoff (2006), Second-best Taxation and Regulation with Differentiated Land Quality, Working paper, University of Missouri.

Baxter, Marianne and Urban J. Jerman (1999), Household Production and the Excess Sensitivity of Consumption to Current Income, *American Economic Review* 89(4), 902-920.

Berkovec, James and Don Fullerton (1992), A General Equilibrium Model of Housing, Taxes, and Portfolio Choice, *Journal of Political Economy* 100(2), 390-429.

Bye, Brita and Turid Åvitsland (2003), The welfare effects of housing taxation in a distorted economy: a general equilibrium analysis, *Economic Modelling* 20, 895-921.

Carey, David and Josette Rabesona (2004), Tax Ratios on labor and Capital Income and on Consumption, in Sorensen, Peter Birch (ed.) *Measuring the Tax Burden on Capital and labor*, The MIT Press, Cambridge, Massachusetts.

Chamley, Christophe (1986), Optimal taxation of capital income in general equilibrium with infinite lives, *Econometrica* 54, 607-22.

Christiansen, Vidar (1984), Which commodity taxes should supplement the income tax?, *Journal of Public Economics* 24, 195–220.

Coleman II, Wilbur John (2000), Welfare and optimum dynamic taxation of consumption and income, *Journal of Public Economics* 76, 1-39.

Cremer, Helmuth and Firouz Gahvari (1998), On Optimal Taxation of Housing, *Journal of Urban Economics* 43, 315-335.

Davis, Morris and Jonathan Heathcote (2005), Housing and the Business Cycle, *International Economic Review*, 46(3), 751-784.

Eerola, Essi and Niku Määttänen (2006), On the Political Economy of Housing's Tax Status, The B.E. Journal in Macroeconomics (Topics), Article 7.

Englund, Peter (2003), Taxing Residential Housing Capital, *Urban Studies*, 40(5-6), 937-952.

Flavin, Marjorie, and Shinobu Nakagawa (2008), A Model of Housing in the Presence of Adjustment Costs: A Structural Interpretation of Habit Persistence, American Economic Review 98, 474–95.

Gahvari, Firouz (1984a), Incidence and efficiency aspects of differential taxation of residential and industrial capital in a growing economy, *Journal of Public Economics* 25, 211-233.

Gahvari, Firouz (1984b), The optimal taxation of housing, *Public Finance*, 39(2), 213-25.

Gahvari, Firouz (1985), Taxation of housing, capital accumulation, and welfare: a study in dynamic tax reform, *Public Finance Quarterly* 13(2), 132–160.

Gervais, Martin (2002), Housing Taxation and Capital Accumulation, *Journal of Monetary Economics* 49(7), 1461-1489.

Gomme, Paul, Finn E. Kydland, and Peter Rupert (2001), Home Production Meets Time to Build, *Journal of Political Economy* 109(5), 1115-1131.

Greenwood, Jeremy and Zvi Hercowitz (1991), The Allocation of Capital and Time over the Business Cycle, *Journal of Political Economy* 99(6), 1188-1214.

Greenwood, Jeremy, Richard Rogerson and Randall Wright (1995), Household Production in Real Business Cycle Theory, in Cooley, Thomas (ed.) *Frontiers of Business Cycle Research*, Princeton University Press.

Hendershott, Patric H. and Michael White (2000), Taxing and Subsidizing Housing Investment: The Rise and Fall of Housing's Favored Status, *Journal of Housing Research* 11(2), 257-275.

Hendershott, P.H. and Y. Won (1992), Introducing Risky Housing and Endogenous Tenure Choice into a Portfolio-Based General Equilibrium Model, *Journal of Public Economics* 48, 293-316.

Institute for Fiscal Studies (2004), The IFS Green Budget 2004 (eds. by Robert Chote, Carl Emmerson and Zoë Oldfield).

Judd, Kenneth L. (1985), Redistributive taxation in a simple perfect foresight model, *Journal of Public Economics* 28, 59-83.

Krusell, Per (2002), Time-consistent redistribution, *European Economic Review* 46, 755-769.

Kydland, Finn (1995), Business cycles and aggregate labor market fluctuations, in Cooley, Thomas (ed.) *Frontiers of Business Cycle Research*, Princeton University Press.

Lansing, Kevin J. (1999), Optimal redistributive capital taxation in a neoclassical growth model, *Journal of Public Economics* 73, 423-453.

McGrattan, Ellen R., Richard Rogerson and Randall Wright (1997), An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy, *International Economic Review* 38(2), 267-290.

Poterba, James (1992), Taxation and Housing: Old Questions, New Answers, American Economic Review 82(2), 237-42.

Siegel, Stephan (2005), Consumption-Based Asset Pricing: Durable Goods, Adjustment Costs, and Aggregation, mimeo, University of Washington.

Skinner, Jonathan (1996), The Dynamic Efficiency Cost of Not Taxing Housing, *Journal of Public Economics* 59(3), 397-417.

Slemrod, Joel (1982) Down-payment constraints: tax policy effects in a growing economy with rental and owner-occupied housing, *Public Finance Quarterly* 10(2), 193-217.

Turnovsky, Stephen J. and Toshiyuki Okuyama (1994), Taxes, housing and capital accumulation in a two-sector growing economy, *Journal of Public Economics* 53, 245-267.

# Appendix

In this appendix, we derive Results 1 and 2. Throughout we assume that the upper bounds for the tax rates on the imputed rent and the return to business capital are not binding, that is $\omega = \psi = 0$. For convenience, we drop time indices.

We first prove the following lemma.

**Lemma 1** *If* $\gamma^c = \gamma^h$, *then*

$$\frac{W_h}{W_c} = \frac{u_h}{u_c}.$$

*where*

$$\begin{aligned} W_c &= u_c + \lambda(u_{hc}h + u_{cc}c + u_c + u_{nc}n) && \text{(A1)} \\ W_h &= u_h + \lambda(u_{hh}h + u_h + u_{ch}c + u_{nh}n). && \text{(A2)} \end{aligned}$$

**Proof.** We have to consider different cases depending on the parameter values in the utility function. Note first that for $\sigma \neq 1$, $\gamma^c = \gamma^h \neq 0$, the utility function we employ may be written as

$$
\begin{aligned}
u(c,h,n) &= \frac{\left[\theta^c c^{\gamma^c} + (1-\theta^c)\left(\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}\right)^{\gamma^c/\gamma^h}\right]^{(1-\sigma)/\gamma^c}}{1-\sigma} \\
&\equiv \frac{S\left(c,h,n\right)^{\frac{1-\sigma}{\gamma^c}}}{1-\sigma}
\end{aligned}
$$

where $S\left(c,h,n\right) = \theta^c c^{\gamma^c} + (1-\theta^c)\left(\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}\right)^{\gamma^c/\gamma^h}$. The partial derivatives of the utility function can then be written using the partial derivatives of $S$ as follows:

$$
u_i = \frac{1}{\gamma^c} S^{\varphi} S_i \text{ for } i = c,h,n \tag{A3}
$$

$$
\begin{aligned}
u_{ij} &= \frac{1}{\gamma^c}\varphi S^{\varphi-1} S_i S_j + \frac{1}{\gamma^c} S^{\varphi} S_{ij} = u_i\left(\varphi\frac{S_j}{S} + \frac{S_{ij}}{S_i}\right) \\
&= u_j\left(\varphi\frac{S_i}{S} + \frac{S_{ij}}{S_j}\right) \text{ for } i,j = c,h,n
\end{aligned} \tag{A4}
$$

where $\varphi = \frac{1-\sigma}{\gamma^c} - 1$. The partial derivatives of $S$ are

$$
\begin{aligned}
S_c &= \gamma^c \theta^c c^{\gamma^c - 1} \\
S_{cc} &= S_c\left(\gamma^c - 1\right)c^{-1} \\
S_h &= (1-\theta^c) D^{\gamma^c/\gamma^h - 1} \theta^h h^{\gamma^h - 1} \\
S_{hh} &= S_h\left[\left(\gamma^c - \gamma^h\right) D^{-1}\theta^h h^{\gamma^h - 1} + \left(\gamma^h - 1\right) h^{-1}\right] \\
S_n &= -(1-\theta^c) D^{\gamma^c/\gamma^h - 1}\left(1-\theta^h\right)\left(1-n\right)^{\gamma^h - 1} \\
S_{ch} &= S_{cn} = 0 \\
S_{hn} &= -S_h D^{-1}\left(\gamma^c - \gamma^h\right)\left(1-\theta^h\right)\left(1-n\right)^{\gamma^h - 1}
\end{aligned}
$$

where $D = \theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}$. Plugging the partial derivatives of $u$ into (A1) and

(A2) and rearranging gives:

$$\begin{aligned} W_c &= u_c + \lambda u_c \left( \frac{\varphi F}{S} + 1 + \frac{S_{cc} c}{S_c} + \frac{S_{cn} n}{S_c} + \frac{S_{ch} h}{S_c} \right) \\ W_h &= u_h + \lambda u_h \left( \frac{\varphi F}{S} + 1 + \frac{S_{hh} h}{S_h} + \frac{S_{hn} n}{S_h} + \frac{S_{ch} c}{S_h} \right) \end{aligned}$$

where $F = S_c c + S_n n + S_h h$. Plugging the partial derivatives of $S$ into the above expressions allows us to write:

$$W_c = u_c \left[ 1 + \lambda \left( \frac{\varphi F}{S} + \gamma^c \right) \right] \tag{A5}$$

$$W_h = u_h \left[ 1 + \lambda \left( \frac{\varphi F}{S} + \gamma^h + \left( \gamma^c - \gamma^h \right) \frac{\theta^h h^{\gamma^h} - \left(1 - \theta^h\right)(1-n)^{\gamma^h - 1} n}{\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}} \right) \right] \tag{A6}$$

From equation (A6) it follows that if $\gamma^c = \gamma^h$,

$$W_h = u_h + \lambda u_h \left( \frac{\varphi F}{S} + \gamma^h \right).$$

Therefore if $\gamma^c = \gamma^h$, (A5) and (A6) imply that

$$\frac{W_h}{W_c} = \frac{u_h}{u_c}.$$

Hence, we have proved Lemma 1 for the case where $\sigma \neq 1$ and $\gamma^c = \gamma^h \neq 0$. It is clear that this result also applies when $\sigma = 1$ and $\gamma^c = \gamma^h \neq 0$, because equations (A3)-(A4) and the derivations thereafter remain valid. Consider then the case where $\gamma^c = \gamma^h = 0$ and $\sigma = 1$. The utility function is then

$$u(c, h, n) = \theta^c \log c + (1 - \theta^c)\theta^h \log h + (1 - \theta^c)(1 - \theta^h) \log (1 - n).$$

Plugging the partial derivatives of $u$ into (A1) and (A2) shows that

$$W_c = u_c \text{ and } W_h = u_h$$

and therefore Lemma 1 applies. Finally, consider the case where $\sigma \neq 1$ and $\gamma^c = \gamma^h = 0$. In this case

$$u(c, h, n) = \frac{\left[ c^{\theta^c} h^{\theta^h (1-\theta^c)} (1-n)^{(1-\theta^h)(1-\theta^c)} \right]^{1-\sigma}}{1 - \sigma}$$

and then

$$u_i = \widetilde{S}^{-\sigma}\widetilde{S}_i \text{ for } i = c, h, n$$
$$u_{ij} = -\sigma\widetilde{S}^{-\sigma-1}\widetilde{S}_i\widetilde{S}_j + \widetilde{S}^{-\sigma}\widetilde{S}_{ij} \text{ for } i, j = c, h, n$$

where $\widetilde{S} = c^{\theta^c} h^{\theta^h(1-\theta^c)}(1-n)^{(1-\theta^h)(1-\theta^c)}$. Proceeding as above we can write

$$\begin{aligned}
\widetilde{S}_c &= \theta^c c^{-1}\widetilde{S} \\
\widetilde{S}_h &= \theta^h(1-\theta^c)h^{-1}\widetilde{S} \\
\widetilde{S}_n &= -(1-\theta^h)(1-\theta^c)\,(1-n)^{-1}\,\widetilde{S} \\
\widetilde{S}_{cc} &= \theta^c c^{-1}\widetilde{S}_c - c^{-1}\widetilde{S}_c \\
\widetilde{S}_{hh} &= \left(\theta^h(1-\theta^c)h^{-1}\right)\widetilde{S}_h - h^{-1}\widetilde{S}_h \\
\widetilde{S}_{ch} &= \theta^h(1-\theta^c)h^{-1}\widetilde{S}_c = \theta^c c^{-1}\widetilde{S}_h \\
\widetilde{S}_{cn} &= -(1-\theta^h)(1-\theta^c)\,(1-n)^{-1}\,\widetilde{S}_c \\
\widetilde{S}_{hn} &= -(1-\theta^h)(1-\theta^c)\,(1-n)^{-1}\,\widetilde{S}_h
\end{aligned}$$

Plugging these expressions into (A1) and (A2) we have

$$\begin{aligned}
W_c &= u_c + \lambda u_c\left(\frac{-\sigma\widetilde{F}}{\widetilde{S}} + \theta^c + \theta^h(1-\theta^c) - (1-\theta^h)(1-\theta^c)\,(1-n)^{-1}\,n\right) \text{ and} \\
W_h &= u_h + \lambda u_h\left(\frac{-\sigma\widetilde{F}}{\widetilde{S}} + \theta^c + \theta^h(1-\theta^c) - (1-\theta^h)(1-\theta^c)\,(1-n)^{-1}\,n\right).
\end{aligned}$$

where $\widetilde{F} = \widetilde{S}_c c + \widetilde{S}_n n + \widetilde{S}_h h$. Again, $\frac{W_h}{W_c} = \frac{u_h}{u_c}$. We have now proved Lemma 1. ■

**Proof of Result 1.** Assume now that $\gamma^c = \gamma^h$. By using Lemma 1 we can write equation (21) as

$$\frac{u_h}{u_c} = \delta_h + r.$$

Inserting this expression into (22) gives

$$\begin{aligned}
\delta_h + r &= \frac{(1+\tau^i)\,[R-(1-\delta_h)]}{1+\tau^c} + \frac{\tau^h\,(1+\tau^i)\,r}{1+\tau^c} \\
&\Leftrightarrow \\
\delta_h + r &= \frac{(1+\tau^i)\left[\left(1-\tau^k\right)r + \delta_h\right]}{1+\tau^c} + \frac{\tau^h\,(1+\tau^i)\,r}{1+\tau^c}
\end{aligned}$$

This proves Result 1. ■

In order to prove Result 2 we first prove the following lemma:

**Lemma 2.** If $\gamma^c \neq \gamma^h$, then

$$\frac{W_h}{W_c} = \frac{u_h}{u_c}\frac{G}{C} \tag{A7}$$

where

$$G = 1 + \lambda S_{\cdot}^{-1}\left(S\gamma^h + \varphi F + S\left(\gamma^c - \gamma^h\right)\frac{\theta^h h^{\gamma^h} - \left(1-\theta^h\right)(1-n)^{\gamma^h-1}n}{\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}}\right)$$

and

$$C = 1 + \lambda S_{\cdot}^{-1}\left(S\gamma^c + \varphi F\right).$$

Because $\gamma^c \neq \gamma^h$, it follows that

$$\frac{G}{C} \neq 1 \tag{A8}$$

In addition, when $\sigma = 1$, we have that

$$\begin{aligned} G &> 1 \text{ if } \gamma^h > \gamma^c \text{ and } \gamma^h \geq 0 \\ 0 &< G < 1 \text{ if } \gamma^h < \gamma^c,\ \gamma^h \leq 0, \text{ and } \gamma^h \text{ is not too small.} \end{aligned}$$

**Proof.** Equation (A7) follows directly from equations (A5) and (A6). We use the expressions for partial derivatives for $u$ and $S$ from Lemma 1 in order to write

$$\begin{aligned} \frac{G}{C} &= 1 \Leftrightarrow \\ S\gamma^h + \varphi F + S\left(\gamma^c - \gamma^h\right)\frac{\theta^h h^{\gamma^h} - \left(1-\theta^h\right)(1-n)^{\gamma^h-1}n}{\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}} &= S\gamma^c + \varphi F \Leftrightarrow \\ \left(\gamma^h - \gamma^c\right)\left(\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}\right)^{-1}(1-\theta^h)(1-n)^{\gamma^h}\left(1 + \frac{n}{1-n}\right) &= 0 \end{aligned}$$

Clearly, the above equation is only satisfied when $\gamma^h = \gamma^c$. Hence, $\gamma^c \neq \gamma^h$ implies that $\frac{G}{C} \neq 1$. In order to determine the sign of $G$ note first that if $\sigma = 1$, then $\varphi = -1$. Therefore we can write $G$ as

$$G = 1 + \frac{\lambda}{SD}\left[DS\gamma^h - DF + S\left(\gamma^c - \gamma^h\right)\left(\theta^h h^{\gamma^h} - \left(1-\theta^h\right)(1-n)^{\gamma^h-1}n\right)\right]$$

where $D = \theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}$. We then proceed by simplifying the expression inside the brackets. We first write $F$ as

$$\begin{aligned} F &= S_c c + S_n n + S_h h \\ &= \gamma^c \theta^c c^{\gamma^c - 1} c - \gamma^c (1-\theta^c) D^{\gamma^c/\gamma^h - 1} \left(1-\theta^h\right) (1-n)^{\gamma^h - 1} n + \gamma^c (1-\theta^c) D^{\gamma^c/\gamma^h - 1} \theta^h h^{\gamma^h - 1} h \\ &= \gamma^c \left[\theta^c c^{\gamma^c} + (1-\theta^c) D^{\gamma^c/\gamma^h - 1} \left(\theta^h h^{\gamma^h} - \left(1-\theta^h\right)(1-n)^{\gamma^h - 1} n\right)\right] \end{aligned}$$

After using the above expression for $F$ and the expression for $S$ from Lemma 1 and rearranging terms we get

$$G = 1 + \frac{\lambda(1-\theta^h)(1-n)^{\gamma^h}}{SD} \left(\frac{1}{1-n}\right) \left(\left(\gamma^h - \gamma^c\right) \theta^c c^{\gamma^c} + \gamma^h (1-\theta^c) D^{\gamma^c/\gamma^h}\right) \tag{A9}$$

This means that a sufficient condition for $G > 1$ is that $\gamma^h > \gamma^c$ and $\gamma^h \geq 0$. In addition, $0 < G < 1$ if $\gamma^h < \gamma^c$, $\gamma^h \leq 0$, and $\gamma^h$ is not too small. The exact threshold for $\gamma^h$ depends, among other things, on the value of $\lambda$ (the multiplier of the implementability constraint). This proves Lemma 2. ■

**Proof of Result 2.** Assume now that $\gamma^c \neq \gamma^h$. With $\tau^c = \tau^i$, (21) and (22) read as

$$\frac{W_h}{W_c} = \delta_h + r \text{ and } \frac{u_h}{u_c} = \left(1-\tau^k\right) r + \delta_h + \tau^h r.$$

Plugging these equations into (A7) gives

$$\begin{aligned} \delta_h + r &= \left[\left(1-\tau^k\right) r + \delta_h + \tau^h r\right] \frac{G}{C} \Leftrightarrow \\ \tau^h - \tau^k &= \left(1 + \frac{\delta_h}{r}\right)\left(\frac{C}{G} - 1\right) \end{aligned} \tag{A10}$$

By Lemma 2, $\gamma^h \neq \gamma^c$ implies that $\frac{C}{G} \neq 1$. Therefore, $\tau^h \neq \tau^k$ if $\gamma^h \neq \gamma^c$. This proves part i) of Result 2.

We will prove part ii) of Result 2 by showing that assumptions $\gamma^h > \gamma^c$ and $\gamma^h \geq 0$ imply that $\frac{C}{G} < 1$. By (A10) it then follows that $\tau^h - \tau^k < 0$. By Lemma 2, we know that assumptions $\gamma^h > \gamma^c$ and $\gamma^h \geq 0$ imply that $G > 0$. Therefore

$$\frac{C}{G} < 1 \Leftrightarrow G > C$$

By using the expressions for $F$ and $S$ from Lemma 1 and 2 and rearranging we get the following:

$$\begin{aligned}
G &> C \Leftrightarrow \\
1+\lambda\left(\gamma^h+\frac{\varphi F}{S}+\left(\gamma^c-\gamma^h\right)\frac{\theta^h h^{\gamma^h}-\left(1-\theta^h\right)(1-n)^{\gamma^h-1}n}{\theta^h h^{\gamma^h}+(1-\theta^h)(1-n)^{\gamma^h}}\right) &> 1+\lambda\left(\gamma^c+\frac{\varphi F}{S}\right)\Leftrightarrow \\
\gamma^h+\left(\gamma^c-\gamma^h\right)\frac{\theta^h h^{\gamma^h}-\left(1-\theta^h\right)(1-n)^{\gamma^h-1}n}{\theta^h h^{\gamma^h}+(1-\theta^h)(1-n)^{\gamma^h}} &> \gamma^c\Leftrightarrow \\
\gamma^h-\gamma^c-\left(\gamma^h-\gamma^c\right)\frac{\theta^h h^{\gamma^h}-\left(1-\theta^h\right)(1-n)^{\gamma^h-1}n}{\theta^h h^{\gamma^h}+(1-\theta^h)(1-n)^{\gamma^h}} &> 0\Leftrightarrow \\
\left(\gamma^h-\gamma^c\right)\left[1-\frac{\theta^h h^{\gamma^h}-\left(1-\theta^h\right)(1-n_{t+1})^{\gamma^h-1}n}{\theta^h h^{\gamma^h}+(1-\theta^h)(1-n)^{\gamma^h}}\right] &> 0\Leftrightarrow \\
\left(\gamma^h-\gamma^c\right)(1-\theta^h)(1-n)^{\gamma^h}\left(1+\frac{n}{1+n}\right) &> 0 \qquad \text{(A11)}
\end{aligned}$$

Hence $G > C$. This proves part ii) of Result 2.

We will prove part iii) of Result 2 by showing that assumptions $\gamma^h \leq 0$, $\gamma^h < \gamma^c$ and $\gamma^h$ not too small imply that $\frac{C}{G} > 1$. By (A10), it then follows that $\tau^h - \tau^k > 0$. By Lemma 2, we know that $0 < G < 1$ and therefore

$$\frac{C}{G} > 1 \Leftrightarrow C > G.$$

In addition, from (A11) we know that

$$G > C \Leftrightarrow \left(\gamma^h-\gamma^c\right)(1-\theta^h)(1-n)^{\gamma^h}\left(1+\frac{n}{1+n}\right) > 0$$

Therefore, $C > G$. This proves part iii) of Result 2. ■

**IN VATT WORKING PAPERS SERIES PUBLISHED PUBLICATIONS**

1. Tomi Kyyrä – Pierpaolo Parrotta – Michael Rosholm: The effect of receiving supplementary UI benefits on unemployment duration. Helsinki 2009.
2. Tuomas Pekkarinen – Roope Uusitalo – Sari Kerr: School tracking and development of cognitive skills. Helsinki 2009.