Customer markets and the welfare effects of monetary policy

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Abstract

The welfare cost of inflation in the canonical New Keynesian DSGE model is due to price dispersion. Calibrating the model to match an empirically plausible markup implies very elastic demand curves. As a consequence, even a small dispersion in prices leads to large distortions in households’ allocation of consumption among different goods and substantial welfare losses. The empirical literature suggest, however, that demand adjusts sluggishly to price changes. I introduce customer markets in an otherwise standard New Keynesian model, by assuming that individual households can only optimize their consumption of an individual good at irregular intervals. With this specification, the welfare loss due to price dispersion is significantly smaller. Moreover, output gap stabilization should be assigned a much more important role in the conduct of monetary policy than previously found in the literature.

Keywords: Monetary policy, Customer markets, Welfare, Inflation persistence

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1 Introduction

The New Keynesian DSGE model has become the workhorse model for monetary policy analysis. Derived from microfoundations, the framework is not only suitable for studying how the economy responds to shocks, and the role of various frictions, but it also provides a welfare criterion for evaluating alternative policies. Remarkably, the central bank’s loss function emerging from this class of models takes the same form as previously assumed in the literature, penalizing variations in the output gap and inflation. A fundamental difference, however, is that while the literature has traditionally assigned about equal weight to inflation and output gap stabilization, the welfare criterion derived from microfoundations gives a much higher weight to inflation stabilization.\footnote{See, e.g., the discussion in Woodford (2003) ch. 6.}

While households prefer a balanced consumption basket, inflation causes relative price dispersion between firms, which affects the allocation of consumption among different goods. Calibration of the model to match an empirically plausible markup, implies very elastic demand curves. As a consequence, even a small degree of price dispersion is very costly in welfare terms, as it implies large distortions in households’ allocation of consumption among different goods. This is not consistent with the sluggish behavior of market shares estimated in the customer market literature. Gottfries (2002) and Lundin et al. (2009), for instance, find short-run price elasticities which are smaller than unity.

In this paper I seek to answer the question how the existence of customer markets affects the welfare cost of inflation and optimal monetary policy. The model I have in mind is one where there are costs associated with the acquisition and processing of information about prices, so that households only occasionally reoptimize their allocation of consumption among different goods. For instance, if different goods are sold at different physical locations, households may chose to only compare price quotes across stores infrequently. This means that information about better shopping opportunities diffuses slowly through the economy. This interpretation is the spirit of the original customer market model proposed by Phelps and Winter (1970). But even if households are fully informed about prices, there may be costs associated with the time and cognitive effort
required to optimally allocate consumption between goods. Such frictions, as well as uncertainty about other product attributes, are factors known to give rise to repeat purchase behavior; see Solomon et al. (2006) ch. 8 and references therein. Explicit modelling of information frictions is technically complicated and I therefore resort to a variant of the signaling mechanism proposed in Calvo (1983). I assume that a household allocates consumption among different goods by choosing the relative consumption of each good, i.e., the quantity consumed of the good relative to the total basket consumed. In each period, the household is only allowed to reoptimize for a random fraction of all goods.

The result is a model where a firm’s market share depends on its lagged market share as well as on current and expected future prices. Ravn et al. (2006) and Nakamura and Steinsson (2009) obtain a similar demand formulation by assuming internal deep habits, i.e., that households form habits in the consumption of individual goods. In contrast to those papers, customer markets in this paper are due to frictions that do not affect households’ preferences for different goods. Also, because the sluggishness of demand for individual goods pertains to the infrequent reoptimization of households’ relative consumption, as opposed to the formation of habits in the consumption level, the aggregate demand relation is not affected by the introduction of customer markets. This facilitates the welfare evaluation, as the introduction of customer markets does not change households’ welfare function, when written in terms of variations of the output gap and the dispersion in consumption across different goods.

Because demand is a function of expected future prices, there is a problem of time inconsistency in price setting. Firms would like to promise low future prices, but renege on these promises when the future arrives. I resolve this problem by assuming that firms commit to state contingent price plans. This assumption is unrealistic if taken literally, but it captures the idea that firms can, to a considerable extent, make promises to their customers. Surveys consistently rank implicit contracts as the most important factor for firms when setting prices. See, e.g., Apel et al. (2001), who report evidence from a survey of Swedish firms, and Fabiani et al. (2007), who review the literature for the Euro area. There is also some narrative evidence available that documents the importance of implicit contracts. Young and Levy (2006) provide evidence for implicit contracts in the marketing
I integrate the customer market framework in an otherwise standard New Keynesian staggered price model. The economy is driven by shocks to the efficient interest rate and cost-push shocks. I first consider the case when monetary policy is implemented through a Taylor rule. As it turns out, aggregate dynamics is quite similar, but inflation is more inertial with customer markets. The most important consequence of customer markets is that the welfare criterion changes. In the model with customer markets, the welfare criterion assigns about equal weight to inflation and output gap stabilization. This leads to an optimal monetary policy that involves a lower volatility of the output gap and a substantially higher volatility of inflation. Moreover, the total welfare loss is smaller with customer markets.

The remainder of this paper is organized as follows. The next section describes the model. The welfare criterion is presented in Section 3, the calibration is described in Section 4, and the results from the numerical simulations are shown in Section 5. Section 6 concludes.

# 2 The model

In this section I analyze the decisions of households and firms, and derive a log-linear approximation to the model.

## 2.1 Households

The economy is populated by a large number of households, indexed by $h \in [0, 1]$. A household $h$ derives utility from the consumption of a large number of different goods, indexed $i \in [0, 1]$, according to the aggregator

$$ C^h_i = \left( \int_0^1 (C^h_{it})^{\gamma/\eta} \, di \right)^{\eta/(\eta-1)} $$

(1)
where $C^h_{it}$ denotes the household’s consumption of good $i$. The household’s utility is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma_C} \left( C^h_t \right)^{1-\sigma_C} - \frac{1}{1+\sigma_N} \left( N^h_t \right)^{1+\sigma_N} \right\},$$

(2)

where $\beta \in (0, 1)$ is the subjective discount factor and $N_t^h$ is the number of working hours supplied.

The household’s budget constraint is

$$B^h_t + \int_0^1 C^h_t P_t di = R_{t-1} B^h_{t-1} + W_t N_t^h + \Phi_t,$$

(3)

where $B^h_t$ denotes bond holdings from $t$ to $t+1$, $P_t$ the price of good $i$, $R_t$ the gross nominal interest rate paid off in $t+1$, $W_t$ the nominal wage, and $\Phi_t$ dividends from firm ownership.

The household solves the intertemporal problem of maximizing (2), subject to (1) and (3). However, I impose the additional restriction that the allocation of consumption across different goods is subject to Calvo (1983) style frictions. In each period, the household draws a random fraction $(1-\theta)$ of all goods. For each of these goods, the household is allowed to reoptimize the relative consumption of the good, i.e., the quantity consumed of the good relative to the total basket consumed. For the remaining goods, the fraction remains fixed. The draw is assumed to be uncorrelated both in time and across households.

Letting $\tilde{C}^h_{it} = \frac{C^h_{it}}{C^h_t}$ denote the relative consumption of good $i$, the Lagrangian for the household’s problem is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma_C} \left( C^h_t \right)^{1-\sigma_C} - \frac{1}{1+\sigma_N} \left( N^h_t \right)^{1+\sigma_N} - \lambda^h_t \left[ 1 - \int_0^1 \left( \tilde{C}^h_{it} \right)^{\frac{\sigma-1}{\sigma}} di \right] \right\},$$

(4)

and

$$-\mu^h_t \left[ B^h_t + C^h_t \int_0^1 P_t \tilde{C}^h_{it} di - W_t N^h_t - R_{t-1} B^h_{t-1} - \Phi_t \right].$$
with the associated first order conditions given by:

\begin{align*}
C^h_t : (C^h_t)^{-\sigma_C} - \mu^h_t P_t &= 0, \quad (5) \\
B^h_t : -\mu^h_t + \beta E_t R_t \mu^h_{t+1} &= 0, \quad (6) \\
N^h_t : -(N^h_t)^{\sigma_N} + \mu^h_t W_t &= 0, \quad (7) \\
\tilde{C}^h_{it} : E_t^h \sum_{k=0}^{\infty} (\theta \beta)^k \left[ \lambda^{h}_{t+k} \left( \tilde{C}^h_{it} \right)^{-\frac{1}{\eta}} - \mu^h_{t+k} C^h_{t+k} P_{it+k} \right] = 0. \quad (8)
\end{align*}

where \( P_t \equiv \int_0^1 P_{it} \tilde{C}^h_{it} di \) is an aggregate "price index". The assumptions that households reoptimize for a random fraction of all goods and that there are many goods in the economy imply, because of the law of large numbers, that \( P_t \) is identical across households. This fundamentally simplifies aggregation and ensures that all households choose the same allocations of aggregate consumption, labor supply, and bond holdings. The first order conditions for consumption and bond holdings can be combined to the familiar consumption Euler equation

\[ C^{\sigma_C}_t = \beta E_t R_t \frac{P_t}{P_{t+1}} C^{\sigma_C}_{t+1}, \quad (9) \]

and the first order condition for labor-supply yields

\[ \frac{N^\sigma_N}{C^{\sigma_C}_t} = \frac{W_t}{P_t}. \quad (10) \]

Let the optimal relative consumption of good \( i \), for a household allowed to reoptimize for that good at time \( t \), be denoted \( \tilde{C}^*_i t \). The first order condition for \( \tilde{C}^h_{it} \) yields

\[ \tilde{C}^*_i t = \left( \frac{\Gamma_{it}}{\Gamma_t} \right)^{-\eta}, \quad (11) \]

where

\[ \Gamma_{it} = E_t \sum_{k=0}^{\infty} (\theta \beta)^k \Delta_{t,t+k} C_{t+k} P_{it+k}, \quad (12) \]
is the expectation of a weighted sum of future prices for good \( i \), \( \beta^k \Delta_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right) \)

is the nominal stochastic discount factor, and \( \Gamma_t = \left[ \int_0^1 \Gamma_{it}^{1-\eta} dt \right]^{-\frac{1}{\eta}} \) is an average across firms of expected future prices. When reoptimizing, the household realizes that it will not be able to optimize again for, on average, \((1 - \theta)^{-1}\) periods, and therefore it takes both current and expected future prices of the good into account. In the special case when \( \theta = 0 \) we have that \( \Gamma_{it} = P_{it} \)

and (11) reduces to the the usual demand specification, where relative demand is a function of the current relative price.

Integrating consumption over households yields that total demand for good \( i \) is given by

\[
Y_{it} = \int_0^1 C_{it}^h dh = C_t \int_0^1 \tilde{C}_{it}^h dh
\]

\[
= \theta C_t \int_0^1 \tilde{C}_{it-1}^h dh + (1-\theta) C_t \int_0^1 \tilde{C}_{it}^* dh
\]

\[
= \theta \left( \frac{Y_{t-1}}{C_{t-1}} \right) C_t + (1-\theta) \left( \frac{\Gamma_{it}}{\Gamma_t} \right)^{-\eta} C_t. \tag{13}
\]

Letting \( \bar{Y}_{it} \equiv \frac{Y_{it}}{C_t} \) denote good \( i \)'s market share and dividing through with \( C_t \) in (13) yields

\[
\bar{Y}_{it} = \theta \bar{Y}_{i,t-1} + (1-\theta) \left( \frac{\Gamma_{it}}{\Gamma_t} \right)^{-\eta}. \tag{14}
\]

With customer markets, the market share of firm \( i \) depends both on the market share in the previous period, and on current and expected future prices.

**2.2 Firms**

Good \( i \) is produced by a monopolist with technology

\[
Y_{it} = N_{it}. \tag{15}
\]
Because the firm’s market share is a function of future prices, there is a problem of time inconsistency. The firm would like to promise low future prices, but renege on this promise when the future arrives. I resolve this by assuming that the firm commits to a state contingent price plan. I also assume that the promise was made an infinitely long time ago, so as to prevent the firm from exploiting that expectations are fixed when the promise is announced.\textsuperscript{2} As discussed in the introduction, this is as a stylized way of modelling customer-firm relations.

Price setting frictions are introduced by assuming that prices are set according to the mechanism in Calvo (1983). In each period, the firm is allowed to reoptimize its price with probability \((1 - \alpha)\), and with probability \(\alpha\) the price remains fixed. Given this restriction, the firm’s problem is to maximize its discounted profit stream:

\[
E_0^t \sum_{i=0}^{\infty} \beta^i \Delta_{0,t} [P_{it} Y_{it} - W_t N_{it}],
\]

subject to (14) and (15). The firm’s Lagrangian is

\[
\mathcal{L} = E_0^t \sum_{i=0}^{\infty} \beta^i \Delta_{0,t} \left\{ C_t P_{it} \bar{Y}_{it} - C_t W_t \bar{Y}_{it} + \nu_{it} \left[ \theta \bar{Y}_{it-1} + (1 - \theta) \left( \frac{\Gamma_{it}}{\Gamma_t} \right)^{\eta-1} \bar{Y}_{it} \right] \right. \\
+ \rho_{it} \left[ \Gamma_{it} - \sum_{k=0}^{\infty} (\theta \beta)^k \Delta_{t,t+k} C_{t+k} P_{it+k} \right] \right\},
\]

with the associated first order conditions given by:

\[
\bar{Y}_{it} : \quad \nu_{it} = (P_{it} - W_t) C_t + \theta \beta E_t \Delta_{t,t+1} \nu_{it+1},
\]

\[
\Gamma_{it} : \quad \rho_{it} = \eta (1 - \theta) \left( \frac{\Gamma_{it}}{\Gamma_t} \right)^{\eta-1} \frac{1}{\Gamma_t} \nu_{it},
\]

\[
P_{it} : \quad E_t^{\infty} \sum_{k=0}^{\infty} (\alpha \beta)^k \Delta_{t,t+k} C_{t+k} \left[ \bar{Y}_{t,t+k|t} - \Psi_{t,t+k|t} \right] = 0,
\]

where \(\Psi_{it} = \sum_{k=0}^{\infty} \theta^k \rho_{it-k}\). The multiplier \(\nu_{it}\) is the shadow value of a marginal increase in the firm’s market share. Equation (17) says that the value of a marginal increase in the firm’s market share is the profits generated by the additional customers today, plus the present value of the additional

\textsuperscript{2}This corresponds to the "timeless perspective" in the monetary policy literature.
future profits, from the new customers, given by $\theta \beta E_t \Delta_{t+1} \nu_{t+1}$. The multiplier $\rho_{it}$ is the shadow cost of a marginal increase in $\Gamma_{it}$, the discounted sum of future prices. Equation (18) says that the cost of a marginal increase in $\Gamma_{it}$, is the resulting decrease in the firm’s market share, given by $\eta (1 - \theta) \left( \frac{\Gamma_t}{\tau_t} \right)^{-\eta-1} \frac{1}{\tau_t}$, multiplied by the value of the lost customers, $\nu_{it}$.

Equation (19) says that the additional revenue, $\bar{Y}_{it}$, generated by a marginal price increase should be equalized to the shadow cost of the price increase, $\Psi_{it}$, resulting from the corresponding reduction in the firm’s market share. When setting up its price plan, at the beginning of time, the firm considers how a price change affects its market share at time $t$, captured by $\rho_{it}$. But since households that reoptimize take expectations about future prices into account, the firm also consider how the price change affects its market share in periods before $t$, this is why $\Psi_{it}$ depends on $\rho_{it-1}, \rho_{it-2}, \ldots$. The notation $t + k \mid t$ denotes the value of a variable at time $t + k$, conditional on that the firm’s price was last reoptimized at time $t$. When prices are sticky, the flexible price optimal condition only holds as an expected discounted average over the expected duration of the price.

The first two first order conditions derived above are conceptually identical to those derived by Ravn et al. (2006), who assume that households form habits in the consumption of individual goods, but that the habitual component depends on aggregate demand for that particular good. They obtain a demand formulation of the same form as traditionally assumed in the customer market literature, where demand is a function of demand in the previous period and the current relative price. Because the price at time $t$ has no effect on demand before $t$, the last first condition in their model corresponds to $\bar{Y}_{it} = \rho_{it}$. Under this formulation, firms face a trade-off between capitalizing on, or investing in, its customer stock. By raising its price, a firm reaps a higher revenue from its existing customers, but at the cost of a smaller market share, which, because the customer stock adjusts sluggishly, reduces revenue in future periods. This mechanism is a crucial component also in the model developed in this paper, but the insight that a price increase at time $t$ also reduces demand before $t$, adds another intertemporal aspect to price setting. When firms commit to price plans, this effect counteracts the incentive for firms to exploit their customers.

The parameter $\theta$ can be viewed as a measure of the degree of customer markets. Increasing
the value of $\theta$ has two opposing effects on the pricing decision. On the one hand, a price increase generates more revenue from existing customers, because fewer customers are able to reoptimize. On the other hand, a price increase in period $t$ has a bigger negative effect on sales in the past as well as in the future. If prices are flexible, it turns out that the optimal price is set with a fixed markup over marginal cost:

$$P_{it} = \frac{\eta}{\eta - 1} W_t,$$

which is identical to the optimal price obtained without customer markets, i.e., when $\theta = 0$. The two effects of increasing $\theta$ cancel out when prices are flexible.\(^3\)

When prices are sticky, the firm is only able to equalize the expected average cost and benefit of a price increase/decrease over the expected duration of the price. Hence, outside steady state, a firm’s price will in general deviate from the optimal flexible price. A firm’s optimal price depends on both its market share and the shadow cost of a price increase $\Psi_{it}$. These, in turn, depend on the whole history of previous prices. This implies that different firms, which reoptimize their prices at the same time, will in general not set the same price. Still, adapting the methods developed in Woodford (2005), it is possible to derive a solution to the log-linearized model.

### 2.3 Log-linearization and equilibrium

Let lower case variables denote log-deviations. Log-linearization of the Euler equation in (9) yields the IS-curve

$$x_t = E_t x_{t+1} - \sigma_e^{-1} \left( r_t - E_t \pi_{t+1} - r^e_t \right),$$

where $r^e_t$ is the efficient interest rate, $x_t$ is the gap between the actual rate of output and the efficient rate of output, and $\pi_t = p_t - p_{t-1}$ is the gross rate of inflation. The IS-relation derived here is identical to that obtained in standard New Keynesian model. Aggregate demand is, up to a first-order approximation, not affected by the presence of customer markets. Shocks emanating from preferences and technology are modelled by assuming that the efficient interest rate follows

\(^3\)This result also holds with internal deep habits under commitment, as shown by Nakamura and Steinsson (2009).
the AR(1) process
\[ r_t^e = \rho_r r_{t-1}^e + \varepsilon_t^e, \] (22)
where \( \varepsilon_t^e \) is an i.i.d. process with standard deviation \( \sigma_r \).

In Appendix A I show that inflation dynamics in the model is determined by a modified Phillips curve of the form:
\[
[\pi_t - \theta \pi_{t-1} - \beta E_t (\pi_{t+1} - \theta E_{t-1} \pi_t)] = \omega s_t + \theta \beta E_t [(\pi_{t+1} - \theta \pi_t) - \beta (\pi_{t+2} - \theta \pi_{t+1})],
\] (23)
where \( \omega = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \zeta \) and \( s_t = (\sigma_C + \sigma_N) x_t \) is aggregate real marginal cost. The introduction of customer markets adds various lags and leads of inflation, and also the previous period’s expectation about current inflation, to the New Keynesian Phillips curve. Moreover, the slope of the Phillips curve depends on \( \zeta \), which is a function of the structural parameters; numerical simulations establish that \( \zeta \) is a decreasing function of \( \theta \).

The Phillips curve can be written on a more familiar form by taking expectations dated at \( t-1 \) on both sides of (23) and solving forward to obtain
\[
E_{t-1} \pi_t = \theta \pi_{t-1} + \left( \frac{\omega}{1-\theta} \right) E_{t-1} \sum_{k=0}^{\infty} \beta^k \left( 1 - \theta^{k+1} \right) s_{t+k}.
\] (24)
Hence, inflation at time \( t \) is given by
\[
\pi_t = \theta \pi_{t-1} + \left( \frac{\omega}{1-\theta} \right) E_{t-1} \sum_{k=0}^{\infty} \beta^k \left( 1 - \theta^{k+1} \right) s_{t+k} + e_t,
\] (25)
where \( e_t = \pi_t - E_{t-1} \pi_t \) is the forecast error of inflation between \( t-1 \) and \( t \).

Without customer markets, setting \( \theta = 0 \), inflation is only a function of the discounted sum of the previous period’s expectation of future marginal costs and the expectation error, which, by construction, is independent of inflation prior to \( t \). With customer markets, inflation also depends on past inflation. Remarkably, the coefficient on the backward-looking component of inflation coincides with the backward-looking component of the firms’ market share equation. Hence, the sluggish
adjustment of market shares, with customer markets, directly translates into inflation persistence. Moreover, while inflation still depends on the discounted sum of expected future marginal costs, marginal costs in the near future are weighted down.

Neither aggregate demand nor the optimal flexible price depend on \( \theta \). Customer markets only affect aggregate dynamics by interacting with price stickiness. It may appear counterintuitive that customer markets imply a different Phillips curve, given that the flexible price equilibrium is not affected by customer markets. The reason is that when prices are sticky, the optimal price is, in addition to movements in marginal cost, also affected by movements in the market share. This explains why inflation is persistent. Consider for instance a monetary injection. Those firms, that are unable to adjust in the wake of the shock, will be stuck with low relative prices and thus accumulate large customer stocks, and therefore have big incentives to raise their prices when given the opportunity to reoptimize. As a consequence, many firms will set positive relative prices, even when the increase in marginal cost has receded. Because inflation, as in the standard New Keynesian model, is proportional to the average relative optimal price, this contributes positively to inflation. It also explains why marginal costs in the near future are weighted down. The failure to track a present-day temporary increase in marginal cost only affects a firm’s profit today, while the negative effect of having a too low market share, because it adjusts sluggishly, affects profits for many periods. Put differently, a firm contemplating on raising its price, realizes that an increase in the price today reduces future optimal prices through its negative effect on the market share, which, because prices are sticky, spills over to a lower optimal price already today.

To make the policy problem non-trivial, I introduce an inefficient time-varying disturbance to marginal cost in the form of a cost-push shock. This captures shocks unrelated to preferences and technology, e.g., markup shocks or time-varying tax-wedges. Written in terms of the output gap, the relation in (23) then reads

\[
\left[ (\pi_t - \theta \pi_{t-1}) - \beta E_t (\pi_{t+1} - \theta E_{t-1} \pi_t) \right] = \omega \left( \sigma_C + \sigma_N \right) x_t + \theta \beta E_t \left[ (\pi_{t+1} - \theta \pi_t) - \beta (\pi_{t+2} - \theta \pi_{t+1}) \right] + \omega u_t,
\]

(26)
where the cost-push $u_t$ follows the AR(1) process

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u,$$

(27)

and $\varepsilon_t^u$ is an $i.i.d.$ process with standard deviation $\sigma_u$.

### 3 Welfare

I evaluate welfare by taking a second order approximation to the "representative household’s" unconditional expected utility flow. Assuming that a subsidy is in place that neutralizes the distortion from monopolistic competition, so that the steady state is efficient, this yields an expression for welfare, as a fraction of steady state consumption, given by

$$-\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \lambda_x x_t^3 + \lambda_\pi \pi_t^2 \right],$$

(28)

where

$$\lambda_x = (\sigma_C + \sigma_N),$$

(29)

$$\lambda_\pi = \frac{\alpha}{(1 - \alpha)(1 - \alpha \beta)} \eta \epsilon.$$

(30)

Here, I have ignored terms independent of policy and third order, or higher, terms. The details of this derivation are in Appendix B. This expression differs from that obtained without customer markets by the parameter $\epsilon$, which is a complicated function of the structural parameters. Numerical simulations establish that $\epsilon$ is a decreasing function of $\theta$. Intuitively, the reduced distortions in households' allocation of consumption among different goods, when $\theta$ is increased, lead to lower welfare losses of price dispersion and a weaker motive for the central bank to stabilize inflation.
4 Calibration

Table 1 summarizes the benchmark calibration, where one period corresponds to one quarter. The value of $\beta$ implies a steady state annual interest rate of about 4 percent. I set $\eta = 11$, which implies a 10 percent steady state markup, and $\alpha = 0.75$, which implies a mean duration of prices of 4 quarters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Household’s subjective discount factor</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>Calvo parameter firms</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.88</td>
<td>Calvo parameter households</td>
</tr>
<tr>
<td>$\eta$</td>
<td>11</td>
<td>Elasticity of substitution between goods</td>
</tr>
<tr>
<td>$\sigma_N$</td>
<td>0.47</td>
<td>Inverse of (Frisch) labor supply elasticity</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>0.16</td>
<td>Inverse of intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.8</td>
<td>Persistence of the shock to efficient interest rate</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>0.8</td>
<td>Persistence of the cost-push shock</td>
</tr>
<tr>
<td>$\sigma^r_r$</td>
<td>0.001</td>
<td>Standard deviation of the shock to efficient interest rate</td>
</tr>
<tr>
<td>$\sigma^u_u$</td>
<td>0.01</td>
<td>Standard deviation of the cost-push shock</td>
</tr>
</tbody>
</table>

The novelty is the calibration of $\theta$. Observing this parameter directly from households’ behavior is hardly a viable option. But we can infer it from the model’s structural relations; as discussed in Section 2.3, the backward-looking component of both the firms’ market share equation and inflation is equal to $\theta$. Fortunately, such estimates are readily available and they also imply a similar calibration. Gottfries (2002) estimates, when the full sample is used, the backward-looking component of his market share equation to be in the range of 0.91 to 0.92. The baseline estimates in Rudd and Whelan (2005) of the backward-looking component of inflation are in the range of 0.79 to 0.91; the lower estimates are obtained when a measure of marginal cost is used instead of the output gap. I consider $\theta = 0.88$ as a reasonable compromise between these values. This implies that a household, on average, reoptimizes the relative consumption of a good every eight quarters.

The long run price elasticity is $\eta$; over time a permanent price change has the same effect on demand as without customer markets. The short run price elasticity is not uniquely defined since it depends on what happens to expectations about future prices. To get a sense for the magnitude of this parameter, I calculate the implied price elasticity for two special cases: when a price change
is completely transitory and when it is permanent. In the former case the short run price elasticity is $\eta (1 - \theta) (1 - \theta \beta)$, and in the latter case it is $\eta (1 - \theta)$. For the value of $\theta$ assumed here, these two cases correspond to a short run price elasticity of 0.17 and 1.32 respectively. As a comparison, Gottfries (2002) estimates a short run (within quarter) price elasticity of 0.22; the corresponding estimate in Lundin et al. (2009) is 0.13.

Following Rotemberg and Woodford (1998), I set $\sigma_C = 0.16$. Then, setting $\sigma_N = 0.47$ gives that $\omega (\sigma_C + \sigma_N)$, the slope of the Phillips curve in (26), without customer markets is 0.0541. This is in the plausible range of estimates usually obtained in the literature, see, e.g., Linde (2005). With customer markets the slope decreases to 0.0110. It is, however, uncertain how the additional lags and leads of inflation in (26) would influence estimation of $\omega$.

The standard deviations of the shocks are calibrated so that the responses to shocks are of reasonable magnitudes (c.f. Smets and Wouters (2007)).

5 Results

I first study how aggregate dynamics is affected by customer markets, assuming that monetary policy is implemented through a Taylor rule. Then, I analyze how optimal policy and welfare are affected by customer markets.

5.1 Monetary policy under a Taylor rule

I first assume that monetary policy is implemented through a Taylor rule on the form

$$r_t = (1 - 0.8) (1.5 \pi_t + 0.1 x_t) + 0.8 r_{t-1}.$$ (31)

These coefficients are roughly consistent with empirical evidence for the Greenspan era, see, e.g., Taylor (1999). The top row of Figure 1 plots the responses of the output gap, inflation, and the nominal interest rate to a one standard deviation innovation to the efficient interest rate.$^4$ The

$^4$For all simulations, the Dynare software, available at http://www.cepremap.cnrs.fr/dynare/, has been used.
solid lines show the response in an economy with customer markets and the dashed lines show an economy without customer markets. Note that the central bank could in principle completely offset the shock, by letting the nominal interest rate perfectly track the rise in the efficient rate. However, the equilibrium interest rate response implied by the Taylor rule is too small to attain this, which triggers a boom.

The bottom row of Figure 1 plots the responses of the output gap, inflation, and the nominal interest rate to a one standard deviation innovation to the cost-push shock. In this case the central bank is unable to simultaneously stabilize the output gap and inflation. The central bank could completely stabilize the output gap, by letting the nominal rate track the efficient interest rate, but such a policy would not stabilize inflation. Since the Taylor rule prescribes a heavy weight on inflation stabilization, the central bank drives down the output gap in order to stabilize inflation,
leaning strongly against the wind.

The introduction of customer markets does not radically change business cycle dynamics, but, as conjectured above, inflation is more inertial. The impulse response of inflation is both lower on impact and more persistent.\(^5\) As discussed in Section 2.3, this is a consequence of that a firm’s optimal price is, in addition to movements in marginal cost, also affected by movements in its market share. To illustrate this, Figure 2 plots prices and market shares for different cohorts of price setters, following the shock to the efficient interest rate, in the economy with customer markets. Also plotted are the counterfactual price that would have been set by a New Keynesian (NK) firm that follows the price setting rule implied by the standard model without customer markets, i.e., sets its price as a discounted average of expected future marginal costs, and the price level. The left most panel in the top row plots a benchmark case with flexible price firms. The other panels plot sticky price firms, but with different ex post realizations of the Calvo signal; each firm is assumed to be allowed to reoptimize its price the first time after the shock at \(t = k - 1\) and thereafter, corresponding to the average duration of prices, every four quarters.

With flexible prices, both the NK firm and the customer market (CM) firm (c.f. equation (20)) set their prices to track the increase in nominal marginal cost. Despite the huge increase in the CM firm’s relative price, the decline in its market share is modest; because the high price is temporary and future prices are low, the discounted sum of expected future prices, which is important for the market share, only increases a little. Now consider the case when \(k = 1\). Prices are set based on the assumption that they will no be reoptimized again for, on average, four quarters. Because marginal cost is falling, the NK firm sets its price lower than the flexible price. The CM firm sets its price even lower; the risk of getting stuck with the price for many periods, and end up with a very low market share, has a counteracting effect on the firm’s price, as discussed in Section 2.3. On the aggregate level, this translates into the impulse response of inflation being smaller on impact with customer markets.

For \(k > 1\), the CM firms’ market shares, because their prices are fixed when the shock occurs, initially rises. When given the opportunity to reoptimize, they therefore have bigger incentives to

---

\(^5\) As a different measure, the first order autocorrelation of inflation increases from 0.60 to 0.78, while remaining virtually unchanged for the output gap and the nominal interest rate.
Figure 2: Price and market share dynamics in the economy with customer markets, following a shock to the efficient interest rate.

raise their prices than the NK firms, who are only concerned about current and future marginal costs. This is most evident in the bottom row of Figure 2. The NK firms adjust their prices gradually toward the new steady state level, never setting prices above the aggregate price level. The CM firms, on the other hand, set their prices substantially higher than the aggregate price level when given the opportunity to reoptimize, thus raising their relative prices and contributing positively to inflation. On the aggregate level, it is this mechanism that translates into the impulse response of inflation being more persistent with customer markets.

5.2 Optimal monetary policy

It was shown above that for a given policy, the main consequence of customer markets pertains to the dynamic response of inflation. I now turn to the question how customer markets affect
the conduct of optimal policy. The central bank’s problem is to choose a sequence \( \{x_t, \pi_t\}_{t=0}^{\infty} \) to minimize the discounted sum of normalized period loss functions:

\[
L = \pi_t^2 + \lambda x_t^2,
\]

where \( \lambda = \frac{\lambda}{\lambda} \) is the relative weight on output gap stabilization, subject to the model’s equilibrium relations given by (21) and (26).

The baseline calibration of \( \theta = 0.88 \) implies that the annualized value of \( \lambda \) is 1.0374, as opposed to 0.0787 without customer markets. In the standard model, inflation stabilization is much more important than output gap stabilization: the elastic demand curves required to match a plausible markup make price dispersion very costly in welfare terms, leading to a strong motive for the central bank to stabilize inflation. This is in stark contrast to the previous literature that usually assigned equal weight to inflation and output gap stabilization. With customer markets, however, the reduced distortions in households’ allocation of consumption among different goods decrease the welfare cost of inflation, and make output gap stabilization about as important as inflation stabilization.

The top row of Figure 3 plots the responses of the output gap, inflation and the nominal interest rate to a one standard deviation innovation to the cost-push shock. The solid lines correspond to an economy with customer markets and the dashed lines to an economy without customer markets. There is a hump-shaped decline in the output gap in both economies, but optimal policy is more gradual with customer markets, resulting in a less intense, but more prolonged, bust. The central bank finds it optimal to accommodate some of the inflationary pressure arising due to the cost-push shock. It is well known in the literature that the central bank, in this class of models, anchors inflation expectations by committing to reverting the price level back to its preshock level. This result carries over to the customer market model. After the initial rise in inflation, the central bank therefore keeps inflation negative for some time. Both the degree of accommodation, and as a result the subsequent drop in inflation, is larger and more persistent with customer markets.

To implement the equilibrium allocation, the central bank engineers an initial decline, somewhat
Figure 3: Impulse responses of the output gap, inflation, and the nominal rate to a cost-push shock under optimal policy. Note: Top row is policy evaluated under the welfare criterion implied by the respective model. Bottom row is policy in both models evaluated under the welfare criterion obtained without customer markets ($\lambda = 0.0786$).

smaller with customer markets, in the real interest rate. Without customer markets, the real rate rises above its steady level already in the second period after the shock. With customer markets, reflecting the more gradual response in the output gap, the real rate remains below its steady state level for an additional four quarters. As a consequence of the differences in the responses of the real rate and inflation, the nominal rate response is reversed in the two economies. Without customer markets, the central bank initially lowers the nominal rate and subsequently tightens its policy. With customer markets, the central bank initially raises the nominal rate and then gradually loosens its policy.

Customer markets imply an optimal policy that differs substantially from that without customer markets: the central bank is more concerned about deviations in the output gap, but less concerned
about deviations in inflation. How much of the difference in policy can be attributed to the effect of customer markets on firms’ price setting and how much can be attributed to the effect on the welfare criterion? To separate these two effects, I also consider the experiment of imposing the welfare criterion obtained without customer markets ($\lambda = 0.0787$) in both models; this comparison is plotted in the bottom row of Figure 3. The impulse responses are very similar, suggesting that the main difference in optimal policy is due to the change in welfare criterion.

This is also supported by the efficient policy frontier in Figure 4. The solid line plots the central bank’s optimal trade-off between (annualized) inflation and output gap stabilization with customer markets for different values of $\lambda$; the different cases discussed above are also marked out. Again, when the same weight on output gap stabilization is used in both models, the implied policies are very similar. Ultimately, the effect of introducing customer markets is a decrease in the volatility of the output gap of about 40 percent, but there is an almost eightfold increase in the volatility of inflation.
5.3 Welfare

In the previous section, it was shown that customer markets have a significant impact on how optimal monetary policy should be conducted. In this section, I evaluate the implications for welfare of different policies. The dotted line in Figure 5 plots the period welfare loss, in percent of steady state consumption, for different values of $\theta$ when policy is optimal. The welfare loss is a decreasing function of $\theta$. As $\theta$ approaches unity, the welfare cost associated with inflation vanishes as there are no distortions in the allocation of consumption among different goods. Consequently, in this case, there is no trade-off between stabilizing the output gap and inflation, and the efficient allocation can be obtained. The solid line corresponds to the baseline Taylor rule. The reduction in the welfare loss as $\theta$ increases is initially more rapid than under optimal policy, but levels out substantially for values of $\theta$ larger than 0.9. In this region, the gain from inflation stabilization is small and a Taylor rule with a small weight on output gap stabilization is unsuccessful in further
reducing the welfare loss.

I also consider two alternative Taylor rules. The first rule, corresponding to the dashed line in Figure 5, sets, keeping the remaining parameters at their baseline values, the weight on inflation stabilization to 5. This rule outperforms the other Taylor rules without customer markets. However, the reduction in the welfare loss as \( \theta \) increases is mediocre and for values of \( \theta \) close to one, as a result of its focus on inflation stabilization, it performs the worst. The second rule, corresponding to the line with alternating dashes and dots in Figure 5, sets, keeping the remaining parameters at their baseline values, the weight on output gap stabilization to 0.3. This rule performs very poorly without customer markets. However, as \( \theta \) increases its performance improves radically and it performs better than the other Taylor rules for large values of \( \theta \), when the gain from stabilizing the output gap is large and the loss from inflation volatility is small.

This section confirms the basic premise of the paper; the reduced distortions in households’ allocation of consumption among different goods, as a result of customer markets, also reduce the welfare loss. Reflecting the weights in the welfare criterion, those policies that focus on output gap stabilization, perform very poorly without customer markets, where stabilizing inflation is important, but dominate policies focusing on inflation stabilization when the economy is characterized by a high degree of customer markets, where stabilizing the output gap is more important.

6 Conclusion

In the standard New Keynesian model, price dispersion is associated with large welfare losses as it implies large distortions in households’ allocation of consumption among different goods. I introduce customer markets by assuming that individual households can only optimize their consumption of an individual good at irregular intervals. The resulting sluggish behavior of market shares reduces the welfare loss of staggered price setting. It also changes the nature of optimal monetary policy, implying that a much higher weight should be given to output gap stabilization than previously found in the literature.

The model is kept simple, but the results should carry over to a more general model. For instance,
if firms have quasi-fixed capital stocks and upward sloping marginal cost curves, distortions in the allocation of production between firms are also a source for welfare losses. The sluggish behavior of demand with customer markets would reduce these losses, reinforcing the arguments laid out in this paper.

In the literature, inflation inertia is usually generated by imposing generalized price indexation schemes. Besides the ad hoc nature of such schemes, they also have the counterfactual implication that all prices are adjusted, albeit not optimally, at all times. The customer market model developed in this paper displays an endogenous inertial response of inflation. Firms that reoptimize at the same time do not, because of market share dynamics, set the same price. Following an increase in marginal cost, those firms that are late to adjust, and therefore have accumulated a large customer stock, will set positive relative prices, even when the increase in marginal cost has receded. It is this behavior of prices that generates inflation inertia. Further investigation of this mechanism is an interesting venue for future research.
References


Appendices

A Solving for aggregate inflation dynamics

Log-linearizing the first order conditions in (17) and (18), these can be combined to yield

\[(z_{it} - \theta z_{it-1}) = -\kappa (\tilde{p}_{it} - s_t) + \theta \beta (E_t^z z_{it+1} - \theta z_{it}) , \quad (A.1)\]

where \(z_{it} = \tilde{y}_{it} - \psi_{it}, \tilde{p}_{it}\) is the firm’s relative price, and \(\kappa = (\eta - 1)(1 - \theta)(1 - \theta \beta)\). In addition, log-linearizing the first order condition for price setting in (19) gives

\[E_t^i \sum_{k=0}^{\infty} (a \beta)^k z_{i,t+k|t} = 0. \quad (A.2)\]

Let \(\tilde{z}_{it} = z_{it} - z_t\), where \(z_t = \int_0^1 z_{it} dt\), and write (A.1) as

\[E_t^i [Q (L) \tilde{z}_{it+1}] = \kappa \tilde{p}_{it} , \quad (A.3)\]

where

\[Q (L) = \theta \beta - (1 + \theta^2 \beta) L + \theta L^2\]

and \(L\) is the lag operator.

Let \(\tilde{p}_{it}\) denote firm \(i\)'s optimal relative reset price. I guess that the law of motion for \(\tilde{z}_{it}\) and \(\tilde{p}_{it}\) is of the form

\[\tilde{z}_{it} = A \tilde{z}_{it-1} + B \tilde{p}_{it}, \quad (A.4)\]

\[\tilde{p}_{it}^* = \tilde{p}_t^* + C \tilde{z}_{it-1}, \quad (A.5)\]

where \(\tilde{p}_t^*\) is a function of only aggregate variables. The assumption that reoptimizing firms are
drawn with random probability further implies that

$$\tilde{p}_t^* = \frac{\alpha}{1 - \alpha} \tau_t.$$  \hfill (A.6)

Substituting (A.4), (A.5), and (A.6) into (A.3), yields

$$\left[(1 + \beta \theta^2) A - \theta (1 + \beta A^2) - \theta \beta (1 - \alpha) ABC \right] \tilde{z}_{it-1}$$

$$+ \left[(1 + \theta^2 \beta) B + \kappa - \theta \beta AB - \theta \beta B [\alpha + (1 - \alpha) CB] \right] \tilde{p}_{it} = 0.$$

Since this equation must hold for all possible values of \(z_{it-1}\) and \(\tilde{p}_{it}\), the coefficients must be zero, i.e.,

$$\left[(1 + \beta \theta^2) A - \theta (1 + \beta A^2) - \theta \beta (1 - \alpha) ABC \right] = 0 \quad (A.7)$$

$$\left[(1 + \theta^2 \beta) B + \kappa - \theta \beta AB - \theta \beta B [\alpha + (1 - \alpha) CB] \right] = 0 \quad (A.8)$$

Next, write the first order condition for price setting as

$$E_t \sum_{k=0}^{\infty} (a \beta)^k (\tilde{z}_{i,t+k} + z_{t+k}) = 0.$$  \hfill (A.9)

Substituting (A.4) into (A.9), taking into account that only states in which the firm is unable to reoptimize are relevant, yields

$$\tilde{p}_{it}^* = E_t \sum_{k=1}^{\infty} (a \beta)^k \pi_{t+k} - \frac{(1 - \alpha \beta)(1 - \alpha \beta A)}{B} E_t \sum_{k=0}^{\infty} (a \beta)^k z_{t+k} - \frac{(1 - \alpha \beta) A}{B} \tilde{z}_{it-1},$$  \hfill (A.10)

which is on the form posited in (A.5) with

$$\tilde{p}_{it}^* = E_t \sum_{k=1}^{\infty} (a \beta)^k \pi_{t+k} - \frac{(1 - \alpha \beta)(1 - \alpha \beta A)}{B} E_t \sum_{k=0}^{\infty} (a \beta)^k z_{t+k}$$  \hfill (A.11)
and

\[ C = - \frac{(1 - \alpha \beta) A}{B}. \]  \hspace{1cm} (A.12)

Rewriting (A.11) recursively and imposing (A.6), yields

\[ \pi_t = \frac{(1 - \alpha \beta) (1 - \alpha)}{\alpha} \theta s_t + \beta E_t \pi_{t+1}, \]  \hspace{1cm} (A.13)

where \( \theta = -\frac{(1 - \alpha \beta A)}{B} \) and

\[ (z_t - \theta z_{t-1}) = \kappa s_t + \theta \beta (E_t z_{t+1} - \theta z_t). \]  \hspace{1cm} (A.14)

Substituting (A.13) into (A.14), yields

\[ [(\pi_t - \theta \pi_{t-1}) - \beta E_t (\pi_{t+1} - \theta E_{t-1} \pi_t)] = \omega s_t + \theta \beta E_t [(\pi_{t+1} - \theta \pi_t) - \beta (\pi_{t+2} - \theta \pi_{t+1})], \]  \hspace{1cm} (A.15)

where \( \omega = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \zeta \) and \( \zeta = \kappa \theta. \)

\section{B A second order approximation to the representative household's utility flow}

A second order approximation to the period utility function

\[ U_t = \frac{1}{1 - \sigma_C} \left( \frac{1}{1 - \sigma_C} - \frac{1}{1 + \sigma_N} \right), \]  \hspace{1cm} (B.1)

yields

\[ \frac{U_t - U}{U_C C} = c_t - n_t + \frac{1}{2} (1 - \sigma_C) c_t^2 - \frac{1}{2} (1 + \sigma_N) n_t^2 + O (\| \xi \|^3). \]  \hspace{1cm} (B.2)

Aggregate labor \( N_t \) is obtained by integrating over individual firms' labor demand:

\[ N_t = \int_0^{1} N_{it} di = \int_0^{1} Y_{it} di = \int_0^{1} \int_0^{1} C_{it}^h dhdi = \int_0^{1} \int_0^{1} C_{it}^h didh = \int_0^{1} C_{it}^h di. \]  \hspace{1cm} (B.3)
The last equality follows from the law of large numbers, implying that $\int_0^1 C_{ti}^h \, dt$ is identical across households. It follows that up to a second order approximation

$$n_t = E_i c_{it}^h + \frac{1}{2} \text{var}_i c_{it}^h + O \left( \|\xi\|^3 \right). \tag{B.4}$$

In addition, the aggregator function in (1) implies that

$$c_t = E_i c_{it}^h + \frac{1}{2} \frac{n_t - 1}{\eta} \text{var}_i c_{it}^h + O \left( \|\xi\|^3 \right). \tag{B.5}$$

Combining these, one obtains

$$c_t = n_t - \frac{1}{2} \frac{1}{\eta} \text{var}_i c_{it}^h + O \left( \|\xi\|^3 \right). \tag{B.6}$$

Substituting this back into (B.2), yields

$$\frac{U_t - U}{U C_C} = -\frac{1}{2} \left( (\sigma_C + \sigma_N) c_t^2 + \eta^{-1} \text{var}_i c_{it}^h \right) + O \left( \|\xi\|^3 \right)$$

$$= -\frac{1}{2} \left( (\sigma_C + \sigma_N) x_t^2 + \eta^{-1} \text{var}_i c_{it}^h \right) + t.i.p. + O \left( \|\xi\|^3 \right). \tag{B.7}$$

The cross-sectional dispersion of the household’s consumption across different goods is given by

$$\text{var}_i c_{it}^h = E_i \left\{ (\log C_{it}^h - E_i \log C_{it}^h)^2 \right\} \quad \tag{B.8}$$

$$= E_i \left\{ (\log C_{it}^h - \log C_t) - (E_i \log C_{it}^h - \log C_t)]^2 \right\} \quad \tag{B.9}$$

$$= E_i \left\{ (\log \bar{C}_{it} - \bar{C}_t)^2 \right\}, \quad \tag{B.10}$$
where $ct \equiv E_i \log C_{it}^h - \log C_i$. It follows that

$$
\begin{align*}
\text{var}_i c_{it}^h &= \text{var}_i \left( \log C_{it}^h - \overline{C}_{t-1} \right) \\
&= E_i \left\{ \left[ \log C_{it}^h - \overline{C}_{t-1} \right]^2 \right\} - \left( E_i \log C_{it}^h - \overline{C}_{t-1} \right)^2 \\
&= \theta E_i \left\{ \left[ \log C_{it-1}^h - \overline{C}_{t-1} \right]^2 \right\} + (1 - \theta) E_i \left\{ \left[ \log C_{it}^h - \overline{C}_{t-1} \right]^2 \right\} - (C_t - \overline{C}_{t-1})^2 \\
&= \theta \text{var}_i c_{it-1}^h + (1 - \theta) \text{var}_i \left( \log C_{it}^h - \overline{C}_{t-1} \right)^2 - (C_t - \overline{C}_{t-1})^2, \quad (B.11)
\end{align*}
$$

where $\gamma_{it} = \log \left( \frac{\gamma_{it}}{\gamma_{i}} \right)$. The aggregator in (1) implies that, up to a first order approximation

$$
\overline{C}_t = 0 + O \left( \|\xi\|^2 \right). \quad (B.12)
$$

Using (B.12) to evaluate (B.11), we obtain

$$
\begin{align*}
\text{var}_i c_{it}^h &= \theta \text{var}_i c_{it-1}^h + (1 - \theta) \eta^2 E_i \left( \overline{\gamma}_{it} \right)^2 + O \left( \|\xi\|^3 \right) \\
&= \theta \text{var}_i c_{it-1}^h + (1 - \theta) \eta^2 \text{var}_i \gamma_{it} + O \left( \|\xi\|^3 \right). \quad (B.13)
\end{align*}
$$

From (12) follows, up to a first order log-linear approximation, that

$$
\overline{\gamma}_{it} = (1 - \theta \beta) \overline{\gamma}_{it} + \theta \beta E_t \overline{\gamma}_{it+1} + O \left( \|\xi\|^2 \right). \quad (B.14)
$$

I guess that the equilibrium law of motion for $\overline{\gamma}_{it}$ is of the form

$$
\overline{\gamma}_{it} = \phi_1 \overline{\gamma}_{it-1} + \phi_2 \overline{\gamma}_{it} + O \left( \|\xi\|^2 \right). \quad (B.15)
$$

Substituting this, together with (A.4), (A.5) and (A.6), into (B.14) and setting the resulting coef-
ficients to zero, yields that the solution to $\phi_1$ and $\phi_2$ must satisfy

$$\phi_1 - \theta \beta A [\phi_1 + \phi_2 (1 - \alpha) C] = 0,$$  \hspace{1cm} (B.16)

$$\phi_2 - (1 - \theta \beta) - \theta \beta [\phi_1 B + \phi_2 [\alpha + (1 - \alpha) CB]] = 0.$$  \hspace{1cm} (B.17)

It follows directly from (A.4) and (B.15) that

$$\text{var}_i z_{it} = A^2 \text{var}_i z_{it-1} + B^2 \text{var}_i p_{it} + 2AB \text{cov}_i (z_{it-1}, p_{it}) + O \left( \|\xi\|^3 \right),$$  \hspace{1cm} (B.18)

$$\text{var}_i \gamma_{it} = \phi_1^2 \text{var}_i z_{it-1} + \phi_2^2 \text{var}_i p_{it} + 2\phi_1 \phi_2 \text{cov}_i (z_{it-1}, p_{it}) + O \left( \|\xi\|^3 \right).$$  \hspace{1cm} (B.19)

The cross-sectional dispersion of prices across firms is given by

$$\text{var}_i p_{it} = E_i \left\{ [\log P_{it} - E_i \log P_{it}]^2 \right\}$$

$$= E_i \left\{ [(\log P_{it} - \log P_t) - (E_i \log P_{it} - \log P_t)]^2 \right\}$$

$$= E_i \left\{ [\log \tilde{P}_{it} - \bar{P}_t]^2 \right\},$$  \hspace{1cm} (B.20)

where $\bar{P}_t \equiv (E_i \log P_{it} - \log P_t)$. It follows that

$$\text{var}_i p_{it} = \text{var}_i \left( \log \tilde{P}_{it} - \bar{P}_{t-1} \right)$$

$$= E_i \left\{ [\log \tilde{P}_{it} - \bar{P}_{t-1}]^2 \right\} - (E_i \log \tilde{P}_{it} - \bar{P}_{t-1})^2$$

$$= \alpha E_i \left\{ [\log \tilde{P}_{it-1} - \bar{P}_{t-1}]^2 \right\} + (1 - \alpha) E_i \left\{ [\log \tilde{P}_{it} - \bar{P}_{t-1}]^2 \right\} - (\bar{P}_t - \bar{P}_{t-1})^2$$

$$= \alpha \text{var}_i p_{it-1} + \alpha \pi_t^2 + (1 - \alpha) E_i \left\{ [\log \tilde{P}_{it} - \bar{P}_{t-1}]^2 \right\} - (\bar{P}_t - \bar{P}_{t-1})^2.$$  \hspace{1cm} (B.21)

The aggregate price index $P_t \equiv \int P_{it} \tilde{C}_{it}^\alpha d\bar{i}$, implies that

$$\bar{P}_t = 0 + O \left( \|\xi\|^2 \right).$$  \hspace{1cm} (B.22)
Using this to evaluate (B.21), one obtains

\[
\text{var}_i p_{it} = \alpha \text{var}_i p_{i(t-1)} + \alpha \pi_i^2 + (1 - \alpha) E_i \{ \tilde{p}_{it}^2 \} + O \left( \| \xi \|^3 \right)
\]

\[
= \alpha \text{var}_i p_{i(t-1)} + \alpha \pi_i^2 + (1 - \alpha) (\tilde{p}_{i}^*)^2 + (1 - \alpha) C^2 E_i \{ \tilde{z}_{i(t-1)} \}^2 + O \left( \| \xi \|^3 \right)
\]

\[
= \alpha \text{var}_i p_{i(t-1)} + \frac{\alpha}{1 - \alpha} \pi_i^2 + (1 - \alpha) C^2 \text{var}_i z_{i(t-1)} + O \left( \| \xi \|^3 \right),
\]

(B.23)

where the second line invokes (A.5) and the last line makes use of (A.6). The cross-sectional covariance of \( \tilde{z}_{i(t-1)} \) and \( \tilde{p}_{it} \) is given by

\[
\text{cov}_i (\tilde{z}_{it-1}, p_{it}) = E_i (\tilde{z}_{it-1}, \tilde{p}_{it}) + O \left( \| \xi \|^3 \right)
\]

\[
= \alpha E_i \left[ (A \tilde{z}_{it-2} + B \tilde{p}_{i(t-1)}) (\tilde{p}_{it-1} - \pi_i) \right] + (1 - \alpha) E_i \left[ \tilde{z}_{it-1} (\tilde{p}_{i}^* + C \tilde{z}_{i(t-1)}) \right] + O \left( \| \xi \|^3 \right)
\]

\[
= \alpha A \text{cov}_i (z_{it-2}, p_{i(t-1)}) + \alpha B \text{var}_i p_{i(t-1)} + (1 - \alpha) C \text{var}_i z_{i(t-1)} + O \left( \| \xi \|^3 \right),
\]

(B.24)

where the second line invokes (A.4) and (A.5).

Exploiting the recursive structure of the variance and covariance terms derived above, forming
discounted sums, yields

\[
\sum_{t=0}^{\infty} \beta^t \text{var}_{c_{it}^h} = \frac{(1-\theta)}{(1-\theta \beta)} \eta^2 \sum_{t=0}^{\infty} \beta^t \text{var}_{\gamma_{it}} + \text{t.i.p} + O \left( \|\xi\|^3 \right), \tag{B.25}
\]

\[
\sum_{t=0}^{\infty} \beta^t \text{var}_{\gamma_{it}} = \phi_1^2 \beta \sum_{t=0}^{\infty} \beta^t \text{var}_{z_{it}} + \phi_2^2 \sum_{t=0}^{\infty} \beta^t \text{var}_{p_{it}} + 2\phi_1 \phi_2 \sum_{t=0}^{\infty} \beta^t \text{cov}_{i} (z_{it-1}, p_{it}) + \text{t.i.p} + O \left( \|\xi\|^3 \right), \tag{B.26}
\]

\[
\sum_{t=0}^{\infty} \beta^t \text{cov}_{i} (z_{it-1}, p_{it}) = \frac{\alpha \beta B}{(1-\alpha \beta A)} \sum_{t=0}^{\infty} \beta^t \text{var}_{p_{it}} + \frac{(1-\alpha) \beta C}{(1-\alpha \beta A)} \sum_{t=0}^{\infty} \beta^t \text{var}_{z_{it}} + \text{t.i.p} + O \left( \|\xi\|^3 \right), \tag{B.27}
\]

\[
\sum_{t=0}^{\infty} \beta^t \text{var}_{z_{it}} = \frac{B^2}{(1-\beta^2 \beta)} \sum_{t=0}^{\infty} \beta^t \text{var}_{p_{it}} + \frac{2AB}{(1-\beta^2 \beta)} \sum_{t=0}^{\infty} \beta^t \text{cov}_{i} (z_{it-1}, p_{it}) + \text{t.i.p} + O \left( \|\xi\|^3 \right), \tag{B.28}
\]

\[
\sum_{t=0}^{\infty} \beta^t \text{var}_{p_{it}} = \frac{\alpha}{(1-\alpha)(1-\alpha \beta)} \sum_{t=0}^{\infty} \beta^t \varphi_i^2 + \frac{(1-\alpha) \beta C^2}{(1-\alpha \beta)} \sum_{t=0}^{\infty} \beta^t \text{var}_{z_{it}} + \text{t.i.p} + O \left( \|\xi\|^3 \right). \tag{B.29}
\]

Similarly, forming a discounted sum of the period loss function in (B.7) and using the sums above to substitute for \( \sum_{t=0}^{\infty} \beta^t \text{var}_{c_{it}^h} \), yields

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{U_t - U}{U_C C} \right) = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( \sigma_C + \sigma_N \right) \varphi_i^2 + \frac{\alpha}{(1-\alpha)(1-\alpha \beta)} \eta \varphi_i^2 \right\} + \text{t.i.p} + O \left( \|\xi\|^3 \right), \tag{B.30}
\]

where

\[
\epsilon = \frac{(1-\theta)}{(1-\theta \beta)} \frac{(1-\alpha \beta)}{\chi}, \tag{B.31}
\]

\[
\chi = \phi_1 \beta \left[ \phi_1 (1-\alpha \beta A) + 2\phi_2 (1-\alpha) \right] \Omega \phi_2 \left[ \phi_2 (1-\alpha \beta A) + 2\phi_1 \alpha \beta B \right], \tag{B.32}
\]

and

\[
\Omega = \frac{(1+\alpha \beta A) B^2}{(1-\alpha \beta A)(1-\beta^2 \beta) - 2(1-\alpha) \beta ABC}. \tag{B.33}
\]

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