The Effects of Openness in a Small Open Monetary Union

Seppo Orjasniemi

April 25, 2010

Abstract

In this paper we present a dynamic stochastic general equilibrium model of a small open monetary union with optimal monetary and fiscal policy. The model is used to study the transmission of country specific shocks and associated exchange rate fluctuations. We show that movements of the monetary union’s terms of trade stabilize the output fluctuations inside the monetary union reducing the need for fiscal stabilization. We also show that, under the optimal policy, fluctuations of the exchange rate and the union wide aggregates are affected by the differences in the degree of nominal rigidities among the monetary union member countries.

Keywords Monetary union, monetary policy, fiscal policy, exchange rate

JEL Classification E52, E62, F41

1 Introduction

In the past few years an extensive amount of work has been done on optimal fiscal and monetary policies in micro-founded models of monetary unions, (MU), with price rigidities. Recent papers include e.g. Beetsma and Jensen (2005), Galí and Monacelli (2008) and Ferrero (2009). Main argument on this field of research is that fiscal policy is needed to stabilize inflation differentials inside the monetary union. Most of these studies are, however, conducted in the context of closed monetary union. Only few studies, including Kirsanova et al (2004), Clausen and Wohltmann (2005) and Campa and González-Mínguez (2006), pay attention on the interaction of monetary union and the
rest of the world. However, the discussion on exchange rate fluctuations and importance of the expenditure switching effects between goods produced in the monetary union and the rest of the world is still incomplete.

The main purpose of this paper is to examine the effects of the terms of trade movements and exchange rate fluctuations, associated with monetary union’s openness to international trade, on the optimal monetary and fiscal policy. Building on a model of closed monetary union by Galí and Monacelli (2008) we construct a model where monetary union consists of two open economies trading with the rest of the world. In particular, we are interested in the economic fluctuations caused by country specific productivity shocks. To extend the analysis of optimal monetary and fiscal policy we follow Bignino (2004) and Beetsma and Jensen (2005) and relax the assumption of symmetric price rigidity among the monetary union members. In our analysis fiscal policy is under full coordination and policy planner is committed to the optimal policy.

As a benchmark, we analyze the optimal monetary and fiscal policy under commitment when the MU countries share the same degree of price rigidity. The optimal policy is to keep union wide levels of inflation, nominal interest rate, output and government spending at their natural levels. The result is that fiscal policies of both monetary union countries are used to stabilize the terms of trade fluctuations inside the MU. The resulting exchange rate fluctuations affect all countries inside the MU, reducing the need for fiscal stabilization. We study also the optimal policy mix when the degree of price rigidity varies across the MU countries. In this case optimal policy cannot keep the union wide variables on their natural levels and the resulting exchange rate fluctuations depend on the origin of the technological shock.

The rest of the paper is organized as follows. In section 2 we lay out the model and the equilibrium dynamics are derived in the section 3. Section 4 analyzes the optimal monetary and fiscal policy mix. The numerical experiments are presented in section 5. Section 6 concludes.

2 A model of a small open monetary union

2.1 Households of the monetary union

We construct a DSGE model with nominal rigidities and monopolistic competition referring to Gali and Monacelli (2005) and Galí and Monacelli (2008). In our model the world consists of continuum of small open economies indexed in the unit interval. The monetary union is formed by two of these small open economies indexed by $H$ and $U$. The countries outside the mon-
etary union are referred as the rest of the world. Since monetary union is of measure zero its economic fluctuations do not affect the economies in the rest of the world. The economies are subject to idiosyncratic shocks on productivity and share the same preferences, technology and market structure. While the representative households share the same preferences, the countries of the monetary union are more open to the international trade than the countries of the rest of the world.

Utility of a representative household in country $i$ depends positively on consumption $C_i$ and public spending $G_i$ and negatively on labor output $L^i$. In period $t$ a representative household maximizes

$$E_0 \sum_{s=0}^{\infty} \beta^{t+s} \left\{ \log C_{t+s}^i + \kappa \log G_{t+s}^i - \frac{(L_{t+s}^i)^{1+\phi}}{1+\phi} \right\}, \quad (1)$$

where $\phi > 0$, $\beta \in (0, 1)$ is a discount factor and $E_0$ denotes the mathematical expectations operator conditional to information available at period 0. The private consumption of the representative household in a MU country $i$ is a composite index defined by:

$$C_i^t = \left( \frac{C_{MU,t}^i}{(1-\alpha)} \right)^{(1-\alpha)} \left( C_{F,t}^i \right)^{\alpha},$$

where $C_{MU,t}^i$ is an index of country $i$’s consumption of goods produced in the rest of the world. Variable $C_{MU,t}^i$ is an index of MU country $i$’s consumption of goods produced in the monetary union and is given by

$$C_{MU,t}^i = 2 \left( C_{i,t}^i \right)^{\frac{1}{2}} \left( C_{j,t}^i \right)^{\frac{1}{2}}, \quad i, j \in \{H, U\}, i \neq j,$$

where $C_{i,t}^i$ is an index of country $i$’s consumption of domestic goods and $C_{j,t}^i$ is an index of country $i$’s consumption of goods produced in the another country of the monetary union. The consumption of goods produced in the rest of the world is given by

$$C_{F,t}^i = \exp \int_0^1 c_{f,t}^i df,$$

where $c_{f,t}^i = \log C_{f,t}^i$.

In the rest of the world the representative household in country $i$ consumes domestically produced goods and imported goods. The consumption index of such household is given by

$$C_i^t = \left( \frac{C_{i,t}^i}{(1-\alpha)} \right)^{(1-\alpha)} \left( C_{F,t}^i \right)^{\alpha},$$
where \( C^i_{F,t} \) now includes also the monetary union. In the equations above the parameter \( \alpha \in [0, 1] \) can be interpreted as a measure of openness. The weight of domestic goods in the utility from private consumption of the households in \( MU \) is \((1 - \alpha) / 2\), while for the households in the rest of the world the weight is \( 1 - \alpha \).

In each economy goods are produced by a continuum of monopolistically competitive firms producing differentiated goods. The consumption of goods originating from country \( i \) is given by

\[
C^i_{i,t} = \left( \int_0^1 C^i_{i,t}(k) \frac{dk}{\epsilon} \right)^{1-\frac{1}{\epsilon}},
\]

where \( \epsilon > 1 \) denotes the elasticity of substitution between the differentiated goods produced in one country and \( k \) denotes the types of goods.

The aggregate consumer price index in a \( MU \) country \( i \) is given by

\[
P^i_{c,t} = \left( P^i_{MU,t} \right)^{1-\alpha} \left( P^i_{F,t} \right)^{\alpha},
\]

where the price index for goods originating from \( MU \) countries is

\[
P^i_{MU,t} = \left( P^i_{i,t} \right)^{\frac{1}{2}} \left( P^i_{j,t} \right)^{\frac{1}{2}}, \quad i, j \in \{H, U\}, j \neq i,
\]

and the subindex \( P^i_{F,t} \), expressed in \( MU \)'s currency, is given by

\[
P^i_{F,t} = \int_0^1 P^i_{j,t} dj, \quad j \in [0, 1]
\]

and is equal for both \( MU \) countries. The price index for goods produced in country \( j \), denoted in country \( i \)'s currency, is given by

\[
P^i_{j,t} = \left( \int_0^1 \left( P^i_{j,t}(k) \right)^{1-\epsilon} dk \right)^{\frac{1}{\epsilon}}, \quad i, j \in [0, 1].
\]

The optimal allocation of expenditures of the representative household in \( MU \) country \( i \) implies the demand functions as follows:

\[
P^i_{c,t}C^i_{c,t} = \frac{1 - \alpha}{2} P^i_{c,t} C^i_{c,t}, \quad P^i_{j,t} C^i_{j,t} = \frac{1 - \alpha}{2} P^i_{j,t} C^i_{j,t},
\]

\[
P^i_{F,t} C^i_{F,t} = \alpha P^i_{c,t} C^i_{c,t},
\]

where \( j \in \{H, U\} \) and \( j \neq i \). The demand for good \( k \) produced in country \( j \) by household in country \( i \) is given by

\[
C^i_{j,t}(k) = \left( \frac{P^i_{j,t}(k)}{P^i_{j,t}} \right)^{-\epsilon} C^i_{j,t}.
\]
Consumption indices with associated price indices for the countries in the rest of the world can be derived analogously.

The exchange rate between monetary union’s currency and the currency of country $i$ in the rest of the world is denoted by $E_{i,t}$. We assume the law of one price to hold. Using equation (2) we may write the consumer price index of goods originating from the rest of the world as

$$P_{F,t} = \int_0^1 (E_{j,t}P_{j,t}^i) \, dj, \quad j \in [0,1].$$

The effective exchange rate of the MU’s currency is defined as

$$E_t = \exp \left( \int_0^1 \log E_{i,t} \, di \right).$$

Using equations (3) we can write the consumption index of a household in MU country $i$ as

$$P_{c,t}^i C_{t}^i = \frac{(1 - \alpha)}{2} P_{t,c}^i C_{t+1}^i + \frac{(1 - \alpha)}{2} P_{j,t}^i C_{j+1}^i + \alpha P_{F,t}^i C_{F,t}^i,$$

where $i, j \in \{H, U\}, i \neq j$. \footnote{For the household $i$ in the rest of the world we can write $P_{c,t}^i C_{t}^i = (1 - \alpha) P_{t,c}^i C_{t+1}^i + \alpha P_{F,t}^i C_{F,t}^i, \ i \in [0,1]$.}

In each period and in each country, households earn labor income $W$. The households also have access to a complete set of state contingent claims traded internationally. Representative household in country $i$ maximizes its utility (1) subject to periodic budget constraint given by

$$P_{c,t}^i C_{t}^i + E_t \{ \Lambda_{t,t+1} B_{t+1}^i \} \leq B_t^i + W_t^i L_t^i - T_t^i,$$

where $T_t^i$ denotes lump-sum taxes. The nominal payoff of the portfolio, including also the shares of firms, held at the end of period $t$ is denoted by $B_{t+1}^i$. The payoff is paid in period $t + 1$ and is discounted by stochastic discount factor $\Lambda_{t,t+1}$. The optimality conditions of the utility maximization problem are given by:

$$L_t^i \phi = W_t^i \frac{C_{t}^i P_{c,t}^i}{C_{t}^i P_{c,t}^i} \quad (4)$$

$$E_t \left\{ C_{i+1}^t P_{c,t+1}^i \right\} = \beta E_t \left\{ \frac{1}{\Lambda_{t,t+1}} \right\} C_{i+1}^t P_{c,t+1}^i \quad (5)$$

Equation (4) is the intratemporal optimality condition, and equation (5) is a conventional Euler equation, where term $\frac{1}{E_t \{ \Lambda_{t,t+1} \}}$ can be defined as the gross nominal interest rate, i.e. $R_t^i = \frac{1}{E_t \{ \Lambda_{t,t+1} \}}$. These optimality conditions hold for each $i \in [0,1]$.\footnote{For the household $i$ in the rest of the world we can write $P_{c,t}^i C_{t}^i = (1 - \alpha) P_{t,c}^i C_{t+1}^i + \alpha P_{F,t}^i C_{F,t}^i, \ i \in [0,1]$.}
2.2 Terms of trade and inflation

The bilateral terms of trade between countries $i$ and $j$ is defined as $S_{i,j,t} = \frac{P_{i,j,t}}{P_{i,i,t}}$, i.e. the price of goods produced in country $j$ in terms of price of goods produced in country $i$. The effective bilateral terms of trade of the country $i$ in the rest of the world is defined by $S_{i} = \frac{P_{i,F,t}}{P_{i,i,t}}$. Denoting the logarithms of variables by lower case letters, we may write the consumer price index in a $MU$ country $i$ as

$$p_{i,c,t} = p_{i,t} + \alpha \Delta s_{i,j,t}^{i}, \quad i,j \in \{H,U\}, i \neq j.$$  

and the consumer price index in country $i$ in the rest of the world as

$$p_{i,c,t} = p_{i,t} + \alpha s_{i,t}^{i}, \quad i \in [0,1].$$  

The domestic producer price inflation in country $i$ is defined as rate of change in the price index of domestically produced goods, $\pi_{i,t}^{d} = \Delta p_{i,t}^{d} = p_{i,t} - p_{i,t-1}$. Using equation (6), the consumer price inflation in country $i$ in the rest of the world can be written as $\pi_{c,t}^{i} = \pi_{i,t}^{d} + \alpha \Delta s_{i,t}^{i}$, and the consumer price inflation in $MU$ country $i$ can be written as

$$\pi_{c,t}^{i} = \pi_{i,t}^{d} + \alpha \Delta s_{i,t}^{i} + \alpha \Delta s_{H,U,t}^{i}, \quad i,j \in \{H,U\}, i \neq j.$$  

The consumer price inflation in the monetary union is defined as the average of country specific inflation levels, i.e. $\pi_{c,t}^{MU} = \frac{1}{2} \left( \pi_{c,t}^{H} + \pi_{c,t}^{U} \right)$. Noting that $s_{H,U,t} = -s_{H,U,t}^{H}$ and $p_{F,F,t}^{H} = p_{F,F,t}^{U} = p_{F,t}$, we may write the union wide consumer price inflation as

$$\pi_{c,t}^{MU} = \frac{1}{2} \left( \pi_{i,t}^{d} + \pi_{i,t}^{U} \right) + \alpha \Delta \log E_{t} + \alpha \pi_{t}^{*},$$  

where $\pi_{t}^{*} = \int_{0}^{1} \pi_{i}^{d} di$.

2.3 International risk sharing

We assume complete markets for state-contingent securities across world economy. Under this assumption an Euler equation analogous to equation (5) holds for households in both $MU$ countries and also in countries in the rest of the world. For a country $i$ in the rest of the world we may write

$$\beta \left( \frac{C_{i,t}}{C_{i,t+1}} \right) \left( \frac{P_{c,t}^{i}}{P_{c,t+1}^{i}} \right) \left( \frac{\xi_{i,t}^{i}}{\xi_{i,t+1}^{i}} \right) = E_{t} \{ \Lambda_{t,t+1} \}.$$  


Combining equations (5) and (8) we may write the following condition between a MU country $H$ and a country $j$ in the rest of the world

$$C^H_t = \vartheta^H_j C^j_t \left( \frac{E^j_t P^j_t}{P^H_{c,t}} \right)^{1-\alpha} \left( S^H_{U,t} \right)^{-(1-\alpha)}^2, \quad \forall t, \quad j \in [0,1]$$  \hspace{1cm} (9)

and for MU country $U$ as

$$C^U_t = \vartheta^U_j C^j_t \left( S^U_{j,t} \right)^{1-\alpha} \left( S^U_{H,t} \right)^{-(1-\alpha)}^2,$$

where $\vartheta^j$ is a constant depending on initial conditions. Integrating equation (9) over $j$, we have an optimal risk-sharing condition as follows:

$$C^H_t = \vartheta_0 C^*_t \left( S^H_{t} \right)^{1-\alpha} \left( S^H_{U,t} \right)^{-(1-\alpha)}^2,$$  \hspace{1cm} (10)

where $C^*_t = \int_0^1 C^j_i di$ is the worldwide aggregate consumption index. As in Chari et al (2002) we assume that $\vartheta_0$ is unity. For the $MU$ countries, logarithmic version of the above equation can be written as

$$c^i_t = c^*_t + (1-\alpha) s^i_t - \frac{(1-\alpha)}{2} s^j_{i,t}, \quad i, j \in H, U, i \neq j,$$

and for the country $i$ in the rest of the world

$$c^i_t = c^*_t + (1-\alpha) s^i_t.$$

Notice that using condition (10) and purchasing power parity inside the $MU$ the consumption levels of the $MU$ countries are equal.

With complete asset markets we may write the uncovered interest rate parity condition between $MU$’s and rest of the world’s interest rates in form

$$\frac{R^u_t}{R^*_t} = \frac{E_t [E_{t+1}]}{E_t},$$

where $R^*_t$ is the world interest rate.

### 2.4 Public sector

Fiscal policy of each country is conducted by local government. As shown in equation the (1), the public spending yields utility to local households. Government of a country $i$ buys only locally produced goods in a CES bundle

$$G^i_t = \left( \int_0^1 G_i(k) \frac{E^i_t}{E^i_{t+1}} dk \right)^{\frac{\gamma_i}{\gamma-1}}.$$
With cost minimization the demand for locally produced good $k$ by government is given by

$$G_{i,t}^i (k) = \left( \frac{P_{i,t}^i (k)}{P_{i,t}^i} \right)^{-\epsilon} G_{i,t}^i. \quad (11)$$

As in Galí and Monacelli (2005) we assume that the government pay an employment subsidy, $\tau^i L^i_t$, to local firms. The public spending and employment subsidy are financed by lump sum taxes $T_{i,t}^i$. The nominal government budget constraint can be written in form

$$T_{i,t}^i = \tau^i L^i_t + P_{i,t}^i G_{i,t}^i.$$ 

### 2.5 Firms

In each country there is a continuum of firms indexed in interval $[0, 1]$. The firms are owned by the local households. The demand for good $k$ produced in a country $i$ by households of country $j$ is given by

$$C_{j,i,t}^j (k) = \left( \frac{P_{i,t}^i (k)}{P_{i,t}^i} \right)^{-\epsilon} C_{j,i,t}^j,$$

and the demand by local government is given above in equation (11).

Each firm in country $i$ produces a differentiated good with linear technology

$$Y_{i,t}^i (k) = A_{i,t}^i L_{i,t}^i (k), \quad i, k \in [0, 1],$$

where the country specific productivity level $A_{i,t}^i$ follows the AR(1) process

$$\log (A_{i,t}^i) = \theta \log (A_{i,t-1}^i) + \epsilon_{i,t}^i \quad \text{with} \quad \theta \in [0, 1].$$

Since production technology is linear and productivity level is common to all producers within a country, the real marginal cost, in terms of locally produced goods, in country $i$ is given by

$$MC_{i,t}^i = \frac{(1 - \tau^i) W_{i,t}^i}{P_{i,t}^i A_{i,t}^i}, \quad (12)$$

where $\tau^i$ is an employment subsidy. Labor is supplied by local households. With linear technology the aggregate labor demand in country $i$ is given simply by

$$L_{i,t}^i = \int_0^1 L_{i,t}^i (k) \, dk = \frac{Y_{i,t}^i Z_{i,t}^i}{A_{i,t}^i}, \quad (13)$$

where $Z_{i,t}^i = \int_0^1 \frac{Y_{i,t}^i (k)}{Y_{i,t}^i} \, dk$ and the aggregate product of country $i$ is defined as

$$Y_{i,t}^i = \left[ \int_0^1 Y_{i,t}^i (k) \, \frac{dk}{\int_0^1 Y_{i,t}^i} \right]^\frac{1}{\epsilon}.$$ 

In logarithmic terms, the first order approximation of aggregate labor demand in country $i$ is thus given as $l_{i,t}^i = y_{i,t}^i - a_{i,t}^i$. 

8
Price setting follows the rule by Calvo (1983). In each period in country \( i \) a fraction \( 0 < 1 - \xi^i < 1 \) of firms are randomly and independently chosen and permitted to choose their prices, while the prices of other firms remain unchanged. The optimal price set in period \( t \) is denoted by \( \hat{P}^i_t \). The consumer price index of goods produced in country \( i \) in period \( t \) is given by

\[
P^i_t = \left( \xi^i \left( P^i_{t-1} \right)^{1-\epsilon} + (1 - \xi^i)(\hat{P}^i_t)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}. \tag{14}
\]

When permitted, a firm chooses its price to maximize the present value of profits over the period when the chosen price is in effect. The optimal price for the monopolistic producer in period \( t \) is given by

\[
\hat{P}^i_t(k) = \frac{\epsilon}{\epsilon - 1} \frac{E_t \left\{ \sum_{j=0}^{\infty} (\beta \xi^i)^j Y^i_{t+j}(k) P^i_{t+j} mc^i_{t+j}(k) \right\}}{E_t \left\{ \sum_{j=0}^{\infty} (\beta \xi^i)^j Y^i_{t+j}(k) \right\}}. \tag{15}
\]

Using log-linear approximations of equations (14) and (15) we can solve for the log-linear price index of traded goods produced in country \( i \)

\[
p^i_t = \frac{\beta}{1 + \beta} E_t \{p^i_{t+1}\} + \frac{1}{1 + \beta} p^i_{t-1} + \frac{(1 - \xi^i)(1 - \beta \xi^i)}{(1 + \beta) \xi^i} \hat{mc}^i_t,
\]

where variables denoted with hat are the percentage deviation from initial steady state value, i.e. \( \hat{x}_t = x_t - x \). From above equation we can solve for domestic producer price inflation:

\[
\pi^i_t = \beta E_t \{\pi^i_{t+1}\} + \frac{(1 - \xi^i)(1 - \beta \xi^i)}{\xi^i} \hat{mc}^i_t. \tag{16}
\]

### 3 Equilibrium dynamics

#### 3.1 Solving for equilibrium dynamics

The Euler equation (5) of a household in country \( i \) can be written in logarithmic terms as

\[
c^i_t = E_t \{c^i_{t+1}\} - \left( r^i_t - E_t \{\pi^i_{c,t+1}\} \right), \tag{17}
\]

where \( \rho = -\log \beta \) and \( r^i_t \) is a logarithm of the nominal gross interest rate of currency used in country \( i \).

Amount of goods produced by firm \( k \) at a country \( i \) is given by

\[
Y^i_t(k) = \left( \frac{P^i_t(k)}{P^i_{t,t}} \right)^{-\epsilon} Y^i_t,
\]

9
where $Y_i^t$ is the aggregate output of a country $i$ defined as

$$Y_i^t = \left( \int_0^1 \left( Y_i^t (k) \right) \right)^\frac{1}{1-\alpha}.$$ 

Market clearing condition for goods originating from a MU country $i$ yields

$$Y_i^t = C_{i,t} + C_{j,t} + \int_0^1 C_{i,f}^t df + G_t^i$$

$$= \left( \frac{P_{i,t}}{P_{t,d}} \right) C_t^i + G_t^i$$

$$= C_t^i \left( S_{j,t}^\alpha \right)^\frac{1-\alpha}{2} \left( S_t^\alpha + G_t^i \right), \quad i, j \in \{ H, U \} , f \in \{ 0, 1 \},$$

(18)

where the second equation is derived using the equations (3) and (9). Market clearing condition for country $i$ in the rest of the world is

$$Y_i^t = C_t^i \left( S_t^\alpha \right) + G_t^i,$$

and for the whole world economy

$$Y_t^* = C_t^* + G_t^*.$$

By denoting the share of public spending in a steady state GDP by $\gamma$, we may approximate equation (18) around a symmetric steady state as

$$\hat{y}_t = (1 - \gamma) \left[ c_t^i + \frac{(1-\alpha)}{2} s_{j,t}^i + \alpha s_t^i \right] + \gamma \hat{g}_t^i, \quad i, j \in \{ H, U \}.$$ 

(19)

Using equations (7), (17) and (19), we can write the dynamic IS equation for a MU country $i$ as

$$\hat{y}_t = E_t \left\{ \hat{y}_{t+1} \right\} - (1 - \gamma) \left( r_t - E_t \left\{ \pi_{t+1}^H \right\} - \rho \right) - \gamma E_t \left\{ \Delta \hat{g}_{t+1}^i \right\}.$$ 

(20)

The same condition holds also for all countries in the rest of the world. We may write the dynamic IS equation for the monetary union as

$$E_t \left\{ \Delta \hat{y}_{t+1}^{MU} \right\} = (1 - \gamma) \left( r_t - \frac{E_t \left\{ \pi_{t+1}^H + \pi_{t+1}^U \right\}}{2} - \rho \right) + \frac{\gamma}{2} E_t \left\{ \Delta \hat{g}_{t+1}^H + \Delta \hat{g}_{t+1}^U \right\}.$$ 

Log-linearized version of equation (12) for real marginal costs of producers in a MU country $i$, can be written as

$$mc_t^i = w_t^i - p_{i,t} + a_t^i + \log (1 - \tau)$$

$$= c_t^i + \phi l_t^i + p_{c,t} - p_{i,t} - a_t^i + \log (1 - \tau) - \log (1 - \kappa)$$

$$= c_t^i + \alpha s_t^i + \frac{(1-\alpha)}{2} s_{j,t}^i - \phi l_t^i - a_t^i + \log (1 - \tau) - \log (1 - \kappa).$$

(21)
where \( i, j \in \{ H, U \} \), \( i \neq j \) and the second equality is derived using the equation (4). Using equation (19) we can write equation (21) as

\[
\bar{mc}_i = \left( \frac{1}{1-\gamma} + \phi \right) \hat{y}_i - \frac{\gamma}{1-\gamma} \hat{g}_i - (1 + \phi) a_i .
\] (22)

Substituting the equation (22) to the equation for domestic inflation, equation (16) we can write the New Keynesian Phillips curve in country \( i \) as:

\[
\pi_i = \beta E_t \left\{ \pi_{i+1} \right\} + \lambda_i (1 + \phi) \left( \hat{y}_i - a_i \right) - \frac{\lambda_i \gamma}{1-\gamma} \left( \hat{g}_i - \hat{y}_i \right) ,
\] (23)

where \( \lambda_i = \frac{(1-\xi^i)(1-\beta^i)}{\xi^i} \).

### 3.2 The efficient allocation with flexible prices

The optimal allocation from the viewpoint of the small open monetary union is a solution to a social planner’s problem of maximizing the average utility of the representative households in the monetary union taking the rest of the world consumption as given. The social planner’s problem is to maximize

\[
V_t = \log C_H^t + \log C_U^t + \kappa \log G_H^t + \kappa \log G_U^t - \frac{(L_H^t)^{1+\phi}}{1+\phi} - \frac{(L_U^t)^{1+\phi}}{1+\phi}
\] (24)

subject to market clearing constraints (18) and technological constraints (13) and risk sharing conditions (10). Using the equations (18) and (10) we may write the consumption level in a \( MU \) country \( i \) as:

\[
C_i^t = (Y^i_H - G_H^t)^{(1-\alpha)\frac{2}{1-\alpha}} (Y^i_U - G_U^t)^{(1-\alpha)\frac{2}{1-\alpha}} (C_i^*)^\alpha , \quad i \in \{ H, U \} .
\] (25)

By plugging equation (25) to equation (24) we may write the first order conditions of the planners problem as

\[
\frac{dV_t}{dG_i^t} = 0 \iff G_i^t = \frac{\kappa}{1-\alpha + \kappa} A_i^t L_i^t , \quad i \in \{ H, U \}
\]

\[
\frac{dV_t}{dL_i^t} = 0 \iff \left( L_i^t \right)^{1+\phi} = 1 - \alpha + \kappa , \quad i \in \{ H, U \} .
\]

The solution to the social planner’s problem is given by pair

\[
(L_i^t, G_i^t) = \left( (1 - \alpha + \kappa)^{\frac{1}{1+\phi}}, \frac{\kappa}{1-\alpha + \kappa} Y^i_t \right) ,
\] (26)
i.e. the employment level is fixed and government consumption is a constant share of output. Notice that higher degree of \( MU \)'s openness to international trade decreases the natural level of employment and increases the share of government consumption in GDP. The optimal allocation for the countries in the rest of the world is equal to the allocation (26).

As shown in Galí and Monacelli (2005), above planners solution can be supported as an equilibrium with flexible prices. By denoting the variables in the presence of flexible prices with over line, we may write the marginal costs in a \( MU \) country \( i \) as

\[
1 - \frac{1}{\epsilon} = \frac{MC^i_t}{1 - \tau} \frac{P^i_c,t}{P^i_t} A^i_t \frac{(L^i_t)^\phi}{Y^i_t} \frac{(S^j_t)^{(1-\alpha)}}{Y^j_t} \]

To guarantee that the flexible price allocation yields the optimal outcome in both \( MU \) countries the governments must set \((1 - \tau)(1 - \alpha)\) to equal \( \frac{\epsilon - 1}{\epsilon} \) and government spending of each \( MU \) country must follow the rule

\[
G^i_t = \kappa \left( 1 - \alpha + \kappa (\mu + a^i_t) \right) \frac{1+\phi}{1+\phi + \phi^i} \]

Using equations (18) and (27) with assumption of full risk sharing we can write the bilateral terms of trade between the \( MU \) countries with flexible prices as

\[
\bar{S}^H_{U,t} = \frac{A^H_i}{A^U_i} \]

and the difference between the producer price inflation rates as

\[
\bar{\pi}^H_t - \bar{\pi}^U_t = -\left( \Delta a^H_t - \Delta a^U_t \right) \]

In logarithmic terms, the output and government spending associated with the flexible price equilibrium and optimal policy are \( \bar{y}^i_t = \log \mu + a^i_t \) and \( \bar{g}^i_t = \log \nu + a^i_t \), where \( \mu = (1 - \alpha + \kappa) \frac{1+\phi}{1+\phi + \phi^i} \) and \( \nu = \kappa \mu^{-\phi} \). Below these variables are referred as natural levels of output and government spending. Notice that the logarithmic steady state values of output and government spending are \( y^i = \log \mu \) and \( g^i = \log \nu \), respectively.
3.3 Dynamics with rigid price setting

We define output and government spending gaps in a MU country $i$ as logarithmic deviation of output and government spending from their natural levels, i.e. $\tilde{y}^*_i = y^*_i - \bar{y}_i$ and $\tilde{g}^*_i = g^*_i - \bar{g}_i$, respectively. The fiscal gap in country $i$ is defined as: $\tilde{f}^*_i = \tilde{g}^*_i - \tilde{y}^*_i = (g^*_i - y^*_i) - \log \nu + \log \mu$. Now we are able to write the New Keynesian Phillips Curve, equation (23), of MU country $i$ in terms of output and fiscal gaps:

$$\pi^i_t = \beta E_t \{\pi^i_{t+1}\} + \lambda^i (1 + \phi) \tilde{y}^i_t - \frac{\kappa \lambda^i}{1 - \alpha} \tilde{f}^i_t.$$  \hfill (28)

Defining the union wide producer price inflation as $\pi^MU_t = \frac{1}{2} \pi^H_t + \frac{1}{2} \pi^U_t$, we can write the monetary union’s NKPC as

$$\pi^MU_t = \beta E_t \{\pi^MU_{t+1}\} + \Lambda^MU (1 + \phi) \tilde{y}^MU_t - \frac{\kappa \lambda^MU}{1 - \alpha} \tilde{f}^MU_t,$$

where $\lambda^MU = \frac{1}{2} (\Lambda^H + \Lambda^U)$. By using equation (20) we can write output gap in a MU country as

$$\tilde{y}^i_t = E_t \{\tilde{y}^{i}_{t+1} - \left( \frac{\kappa}{1 - \alpha} \right) \tilde{f}^{MU}_{t+1} \} - \left( r_t - E_t \{\pi^i_{t+1}\} \right) + \left( \frac{\kappa}{1 - \alpha} \right) \tilde{f}^i_t,$$ \hfill (29)

where $\tilde{r}^i_t$ is the natural rate of interest in a MU country $i$ given by

$$\tilde{r}^i_t = \rho + \left( \frac{1 - \alpha + \kappa}{1 - \alpha} \right) E_t \{\Delta \tilde{y}^i_{t+1} - \frac{\kappa}{1 - \alpha + \kappa} \Delta \tilde{\pi}^i_{t+1} \}$$

$$= \rho + E_t \{\Delta \tilde{y}^i_{t+1}\} = \rho + E_t \{\Delta a^i_{t+1}\}.$$  \hfill (30)

The monetary union’s output gap is given by

$$\tilde{y}^{MU}_{t+1} = E_t \{\tilde{y}^{MU}_{t+1} - \left( \frac{\kappa}{1 - \alpha} \right) \tilde{f}^{MU}_{t+1} \} - \left( r_t - E_t \{\pi^{MU}_{t+1}\} \right) + \left( \frac{\kappa}{1 - \alpha} \right) \tilde{f}^{MU}_t,$$

where $\tilde{r}^{MU}_t = \rho + E_t \{\Delta a^{H}_{t+1} + \Delta a^{U}_{t+1}\}$. With equation (29) we can write the difference between changes of output gaps of the MU countries as

$$\Delta \tilde{y}^H_t - \Delta \tilde{y}^U_t = \frac{\kappa}{1 - \alpha} \left( \Delta \tilde{f}^H_t - \Delta \tilde{f}^U_t \right) - \left[ \left( \pi^H_t - \pi^U_t \right) + \left( \Delta a^H_t - \Delta a^U_t \right) \right].$$  \hfill (31)
4 Optimal monetary and fiscal policy under commitment

4.1 Optimization

We restrict our analysis of the optimal fiscal and monetary policies to a case with full coordination, i.e. monetary and fiscal policy are practiced by a single policymaker to maximize the average welfare of households in the monetary union. The second order Taylor approximation of average welfare of households in the monetary union is derived in appendix (A) and can be written as

\[
V_t = -\Upsilon E_t \sum_{s=0}^{\infty} \sum_{i \in \{H,U\}} \left\{ \frac{\beta t+s}{2} \left[ \frac{\epsilon_i}{\lambda_i} (\pi_{t+s}^i)^2 + \frac{\kappa}{1-\alpha} (\tilde{f}_{t+s}^i)^2 + (1 + \phi) (\tilde{y}_{t+s}^i)^2 \right] \right\} + \text{t.i.p}, \tag{32}
\]

where \( \Upsilon = 1 - \alpha + \kappa \) and the term t.i.p refers to terms that are independent to policy. Notice that utility parameter \( \kappa \) and parameter for openness of the monetary union \( \alpha \) enter the utility function (32) in a weight for fiscal gap and they also affect the level of utility.

The optimal policy can be interpreted as rules for interest rate \( \{r_t\}_{s=0}^{\infty} \) and national fiscal gaps \( \{\tilde{f}_t^H, \tilde{f}_t^U\}_{s=0}^{\infty} \) that maximize (32) subject to the New Keynesian Phillips Curve of the MU (28) and the difference between changes of output gaps of the MU countries (31). Following Galí and Monacelli (2008) we first solve for optimal processes for \( \{\tilde{f}_t^H, \tilde{y}_t^H, \pi_t^H\}_{s=0}^{\infty}, i \in \{H,U\} \), and then solve for path of nominal interest rate that support the optimal process. The first order conditions of the maximization problem are

\[
E_t \left\{ \frac{\epsilon}{\lambda^H} \pi_{t+s}^H - \Delta \psi_{\pi,t+s}^H - \psi_{y,t+s} \right\} = 0, \quad \forall s \geq 0 \tag{33}
\]

\[
E_t \left\{ \frac{\epsilon}{\lambda^U} \pi_{t+s}^U - \Delta \psi_{\pi,t+s}^U + \psi_{y,t+s} \right\} = 0, \quad \forall s \geq 0 \tag{34}
\]

\[
E_t \left\{ \tilde{f}_{t+s}^H - \lambda^H \psi_{\pi,t+s}^H + \psi_{y,t+s} - \beta \psi_{y,t+s+1} \right\} = 0, \quad \forall s \geq 0 \tag{35}
\]

\[
E_t \left\{ \tilde{f}_{t+s}^U - \lambda^U \psi_{\pi,t+s}^U - \psi_{y,t+s} + \beta \psi_{y,t+s+1} \right\} = 0, \quad \forall s \geq 0 \tag{36}
\]

\[
E_t \left\{ (1 + \phi) \tilde{y}_{t+s}^H + \lambda^H (1 + \phi) \psi_{\pi,t+s}^H - \psi_{y,t+s} + \beta \psi_{y,t+s+1} \right\} = 0, \quad \forall s \geq 0 \tag{37}
\]

\[
E_t \left\{ (1 + \phi) \tilde{y}_{t+s}^U + \lambda^U (1 + \phi) \psi_{\pi,t+s}^U + \psi_{y,t+s} - \beta \psi_{y,t+s+1} \right\} = 0, \quad \forall s \geq 0 \tag{38}
\]
where $\psi^{i,\pi}_{t+s}$ are Lagrange multipliers associated with equations (28) and $\psi^{y}_{t+s}$ is associated with equation (31). Using equations (33) and (34) we can write the producer price inflation in the monetary union as

$$E_t \left\{ \pi^{MU}_{t+s} \right\} = \frac{E_t}{2} \left\{ \frac{1}{\epsilon} \left( \lambda^H \Delta \psi^{H,\pi}_{t+s} + \lambda^U \Delta \psi^{U,\pi}_{t+s} \right) + \left( \lambda^H - \lambda^U \right) \psi^{y}_{t+s} \right\}. \quad (39)$$

With first order conditions (35) and (36) we can write the union wide fiscal gap as

$$E_t \left\{ \tilde{f}^{MU}_{t+s} \right\} = \frac{E_t}{2} \left\{ \lambda^H \psi^{H,\pi}_{t+s} + \lambda^U \psi^{U,\pi}_{t+s} \right\}. \quad (40)$$

and using equations (37) and (38) we can write the union wide output gap as

$$E_t \left\{ \tilde{y}^{MU}_{t+s} \right\} = -\frac{E_t}{2} \left\{ \lambda^H \psi^{H,\pi}_{t+s} + \lambda^U \psi^{U,\pi}_{t+s} \right\}. \quad (41)$$

Together the first order conditions (35) and (37), or equivalently (36) and (38), imply that

$$E_t \left\{ \tilde{f}^{i}_{t+s} + (1 + \phi) \tilde{y}^{i}_{t+s} \right\} = -\lambda^i \phi E_t \left\{ \psi^{i,\pi}_{t+s} \right\}. \quad (42)$$

Using the conditions (40) and (41) above, we can see that under optimal policy

$$E_t \left\{ \tilde{f}^{MU}_{t+s} \right\} = -E_t \left\{ \tilde{y}^{MU}_{t+s} \right\}. \quad (43)$$

The rational expectations equilibrium for monetary union satisfies equations (28) and (31) and the first order conditions of the maximization problem for any stochastic processes $\{a^H_{t+s}, a^U_{t+s}\}_{s=0}^{\infty}$. We can write the rational expectations equilibrium as an allocation $\{\pi^{i,\pi}_{t+s}, \tilde{f}^{i}_{t+s}, \tilde{y}^{i}_{t+s}, \psi^{i,\pi}_{t+s}, \psi^{y}_{t+s}\}_{s=0}^{\infty}$, with $i \in \{H, U\}$ and initial condition $\psi^{i,\pi}_{0} = 0$. Using equation (30) we can now write the equilibrium path of nominal interest rate of common currency as

$$r_t = \left( 1 + \frac{K}{1 - \alpha} \right) E_t \left\{ \Delta \tilde{y}^{MU}_{t+1} \right\} + E_t \left\{ \pi^{MU}_{t+1} \right\} + \pi^{MU}_{t}, \quad (44)$$

i.e. the nominal interest rate depends on expected change of union wide output gap, expected union wide producer price inflation and the natural level of interest rate.

Consider now the policy of setting $\pi^{MU}_{t} = \tilde{f}^{MU}_{t} = \tilde{y}^{MU}_{t} = 0$ for every period. Equation (43) implies that under this assumption we have $\psi^{y}_{t} = 0$ in every period. Thus suggested policy is not an equilibrium.
4.2 Symmetric price rigidities

In a special case with symmetric nominal rigidities, i.e. $\lambda^H = \lambda^U$, we have

$$\pi_t^{MU} = -\frac{1}{\epsilon} \Delta \tilde{y}_t^{MU} = \frac{1}{\epsilon} \Delta \tilde{f}_t^{MU}.$$ 

In this case, setting $\pi_t^{MU} = \tilde{f}_t^{MU} = \tilde{y}_t^{MU} = 0$, for every period, maximizes the welfare of households in monetary union\(^2\). With staggered pricing only fraction of producer prices adjust, thus $\psi_{\pi,t} < 0$. By equation (42) we may conclude that setting $\tilde{f}_t = \tilde{y}_t = 0$ for every period is not an equilibrium. Note that under optimal policy we have $\tilde{f}_t^H = -\tilde{f}_t^U$ and $\tilde{y}_t^H = -\tilde{y}_t^U$ implying that also $\pi_t^H = -\pi_t^U$ in every period. In this special case optimal monetary policy is to keep the nominal interest rate of the common currency equal to its natural level.

5 Numerical experiments

In this section we present three numerical experiments. As a benchmark we first study the implications of an unit innovation in the productivity at country $H$ under an assumption of symmetric price rigidities across the $MU$ countries. In the second experiment we study the effect of openness to international trade on the fluctuations caused by country specific shock. Finally in third experiment we study the effects of same shock under asymmetric price rigidities.

5.1 Parametrization

The time frequency of the model is interpreted to be one quarter. For the annual interest rate of four percent the discount factor $\beta$ has a parameter value of .99, which is standard in quarterly business cycle models. As in Galí and Monacelli (2005) the steady state markup is set to be 20 percent, i.e. the value of elasticity of substitution between locally produced goods $\epsilon$ is 6. We also assume that elasticity of labor supply is $\frac{1}{3}$, i.e. $\phi$ has value 3. In the euro area average share of public spending in GDP is approximately 23 percent and the openness to trade, defined as share of imports plus exports of GDP, in the whole euro area is 13 percent. To replicate these estimates we set the parameters $\alpha$ and $\kappa$, to have values .08 and .27, respectively. For the AR(1) process on labor productivity we use the estimates by Galí and Monacelli (2005) and set $\theta$ to have value .66.

\(^2\)This result is common in literature, see for example Galí and Monacelli (2008).
5.2 Symmetric nominal rigidities

First we study the dynamics effects of a productivity shock in $MU$ country $H$ under the optimal policy in the special case with symmetric nominal rigidities in both $MU$ countries. As in Galí and Monacelli (2005) we assume that in each period 25 percent of firms set new prices, i.e. $\xi^i$ has value 0.75. The impulse responses to a unit innovation in $a^H_t$ are presented in Figure 1.

The one percent rise of productivity in country $H$ increases the natural level of output by an identical fraction, while in country $U$ the natural level of output stays unaltered. Rise of productivity in the monetary union also causes a decrease in natural rate of nominal interest rate. The optimal policy is to keep the nominal interest rate on its natural level, thus the nominal interest rate of common currency falls. By uncovered interest rate parity, the fall of the nominal interest rate causes instant depreciation of the common currency. As a result the effective terms of trade in both $MU$ countries increase, inducing an expenditure switching effect from goods produced in
the rest of the world to goods produced in MU.

With increased demand the local producers in country \( U \) are able to increase their prices, while producers in country \( H \) are able to lower their prices with diminished marginal costs. Under the optimal policy, the fall of the producer prices in country \( H \) is balanced by an equal rise of producer prices in country \( U \). As a result, output gap in country \( H \) is negative, i.e. output level does not reach its flexible price level, while in country \( U \) output gap is positive. In both countries these inefficiencies are smoothed by fiscal policy. As a result government spending exceeds its natural level in country \( H \) and the opposite in country \( U \).

5.3 Effect of openness

We study the implications of openness to international trade under symmetric price rigidities. Keeping the values of other parameters constant we examine the differences between closed monetary union and open monetary union by setting degree of openness to have values \( 0, .2, \) and \( .5 \). Impulse responses for unit innovation on the productivity at country \( H \) are presented in Figure 2. As in the case above, under symmetric price rigidities and optimal policy, the union wide variables of output and fiscal gaps and producer price inflation remain at their natural level.

Depreciation of common currency increases the real demand of goods produced in the monetary union by households in both \( MU \) and the rest of the world. This expenditure switching effect decreases the stabilization by government spending. As seen from Figure 2, fluctuations of both fiscal and output gaps are smaller when the monetary union is more open to the international trade.

5.4 Asymmetric rigidities

In the third exercise we study the dynamic effects of a productivity shock in \( MU \) country \( H \) under the optimal policy when the price rigidity is different between the \( MU \) countries. We consider two cases. In the first case \( \xi^H = .75 \) and \( \xi^U = .5, \) i.e. prices are stickier in country \( H \) than in country \( U \). In the second case \( \xi^H = .5 \) and \( \xi^U = .75, \) i.e. prices in country \( H \) are more flexible than in country \( U \). Same parameter values are also used in the analysis by Beetsma and Jensen (2005). The impulse responses for both cases are presented in Figure 3. As noted above, setting \( \pi_t^{MU} = \tilde{f}_t^{MU} = \tilde{y}_t^{MU} = 0 \) for every period, when price rigidities are asymmetric, is not an equilibrium.

Let us first look at the case with less nominal frictions in country \( U \). As in our benchmark case, due to nominal rigidities prices react slowly to the fall
of marginal costs in country $H$. As the prices are more flexible in country $U$, resulting producer price inflation is faster in country $U$ than producer price deflation in country $H$, the resulting union wide inflation is positive. Due to difference in price rigidities, the output in country $U$ is closer to its natural level than output in country $H$, inducing a union wide output gap. As argued above, the resulting union wide fiscal gap is negative of output gap.

In the second case, prices are more rigid in country $U$ than in in country $H$. Now the resulting producer price deflation in country $H$ is faster than producer price inflation in country $U$, resulting a union wide producer price deflation. With more flexible prices, output in country $H$ is closer to its natural level than output in country $U$, thus the union wide output gap is now positive.

In both of the cases considered, increase of productivity in monetary union area causes a decrease in natural rate of nominal interest rate. As shown in equation (44) optimal path of nominal interest rate now depends on expected
union wide inflation and expected change in union wide output gap. In the first case both of these are positive, thus the nominal interest rate is higher than the natural interest rate. In the second case the situation is exactly the opposite. Thus we may conclude that when the planner is maximizing the average utility of the households in MU fluctuations of the nominal interest rate and the exchange rate are driven by the economic fluctuations in the country with less rigid prices.

6 Conclusions

This paper has explored the optimal monetary and fiscal policy mix in a micro founded model of a small open monetary union with price rigidities. The analysis of the optimal monetary and fiscal policy is restricted to full coordination and commitment. As the equilibrium dynamics of the small
open monetary union are analogous to its closed counterpart, the model presented in this paper allows us to study of the exchange rate fluctuations of the common currency and the associated fluctuations of the terms of trade.

We have shown that the openness to international trade decreases the economic fluctuations inside the monetary union. We have also shown that when price rigidities are asymmetric across the countries, the optimal policy allows the union wide variables to deviate from their natural levels.

To derive the second order approximation to the average utility of the small monetary unions household we have restricted the parameter values of the model. Relaxing the assumptions on parameters, e.g. log utility, linear technology, unit elasticity of substitution between goods with different origin, would introduce more interesting dynamics to the model. Also relaxing the assumption of symmetric size and symmetric consumption allocation between the MU countries would bring the model closer to econometric applications. Also some comparison with a model with independent and discretionary fiscal policy could be fruitful.

A Welfare loss function

Using equation (25) we can write the sum of utilities of representative households in the MU as

\[ V_t = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (1 - \alpha) \left[ \log (Y^H_t - G^H_t) + \log (Y^U_t - G^U_t) \right] + 2\alpha c^*_t \right. \]

\[ \left. + \kappa \left( g^H_t + g^U_t \right) - \frac{(L^H_t)^{1+\phi} + (L^U_t)^{1+\phi}}{1+\phi} \right\}. \]

As shown in Appendix of Gali and Monacelli (2008), the second order Taylor approximation of term \( \log (Y^i_t - G^i_t) \) can be written in terms of output and government spending gaps as

\[ \log (Y^i_t - G^i_t) = \frac{1 - \alpha + \kappa}{1 - \alpha} \tilde{y}_t^i - \frac{\kappa}{1 - \alpha} \tilde{g}_t^i - \frac{(1 - \alpha + \kappa)}{2(1 - \alpha)^2} (\tilde{g}_t^i - \tilde{y}_t^i)^2 \]

\[ + \text{t.i.p} + o (\| \cdot \|^3), \]

where t.i.p denotes the terms that are independent of policy and \( o (\| \cdot \|^3) \) represents terms that are of order higher than second, in the bound \( \| \cdot \| \) on the amplitude of productivity shock.

Following Gali and Monacelli (2005) we can write the second order Taylor approximation of term for disutility from labor output about its flexible price

21
level as
\[
\frac{(L_t)^{1+\phi}}{1+\phi} = \left(\frac{L_t}{1+\phi}\right)^{1+\phi} + (L_t)^{1+\phi}(\hat{\varphi}_t + 1 + \frac{1}{2}(1 + \phi)(\hat{y}_t))^2 + o\left(\|a\|^3\right)
\]
\[
= (1 - \alpha + \kappa)(\hat{\varphi}_t + 1 + \frac{1}{2}(1 + \phi)(\hat{y}_t))^2 + t.i.p + o\left(\|a\|^3\right),
\]

where
\[
z_t^i = \log f^i_0 \frac{Y_t^i(k)}{\lambda_t^i} \, dk = f^i_0 \left(\frac{P_i^i(k)}{\lambda_t^i}\right)^{-\epsilon} \, dk.
\]

Collecting the terms above, we can write the second order Taylor approximation of MU’s utility function as
\[
V_t = -(1 - \alpha + \kappa) \sum_{i \in \{H, U\}} \sum_{t=0}^{\infty} \beta^t \left[z_t^i + \frac{\kappa}{2(1-\alpha)} (\hat{f}_t)^2 + \frac{1}{2} (1 + \phi)(\hat{y}_t))^2\right] + t.i.p + o\left(\|a\|^3\right).
\]

**Lemma 1.** \(z_t^i = \frac{\epsilon}{2} \text{var}_k \{p_t^i(k)\} + o\left(\|a\|^2\right).\)

**Proof.** Galí and Monacelli (2005).

**Lemma 2.** \(\sum_{t=0}^{\infty} \beta^t \text{var}_k \{p_t^i(k)\} = \frac{1}{\lambda^i} \sum_{t=0}^{\infty} \beta^t (\pi_t^i)^2,\) where \(\lambda^i = \frac{(1-\epsilon^i)(1-\beta^i)}{\epsilon^i}.\)

**Proof.** Woodford (2003).

Now we can write the second order Taylor approximation of sum of utilities of representative households in the MU as

\[
V_t = -(1 - \alpha + \kappa) \sum_{i \in \{H, U\}} \sum_{t=0}^{\infty} \beta^t \left[\frac{\epsilon}{\lambda^i} (\pi_t^i)^2 + \frac{\kappa}{1-\alpha} (\hat{f}_t)^2 + (1 + \phi)(\hat{y}_t)^2\right] + t.i.p + o\left(\|a\|^3\right).
\]

**References**


