The optimal tax treatment of housing capital in the neoclassical growth model*

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March, 2010

Abstract

Should housing capital be be taxed like other forms of capital? We analyze this question within a version of the neoclassical growth model. We derive the optimal tax treatment of housing capital vis-à-vis business capital allowing for relatively general household preferences. In the first-best, the tax treatment of business and housing capital should always be the same. In the second-best, in contrast, the optimal tax treatment of housing capital depends on the elasticities of substitution between non-housing consumption, housing and leisure. This is because housing taxation may be used to alleviate the distorting effect of taxing labor. As a result, the optimal tax treatment of housing capital may be different from that of business capital. We complement these analytical results with a numerical analysis.

Keywords: Optimal taxation; Dynamic taxation; Housing taxation

JEL codes: H21, E21

*We thank Markus Haavio, Seppo Honkapohja, Karlo Kauko, Erkki Koskela, Henry Ohlsson, Mats Persson, Tuukka Saarimaa, Jouko Vilmunen, Matti Viren and two anonymous referees as well as seminar participants at IIES, HECER, Bank of Finland, and Annual Meetings of the European Economic Association and the Finnish Economic Association for useful comments and suggestions. This research project was initiated during Eerola's visit at the Bank of Finland Research Department. She wishes to thank the Department for its hospitality during her visit. Both authors gratefully acknowledge financial support from the Yrjö Jahnsson Foundation.

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1 Introduction

The tax treatment of housing is an important fiscal question because housing wealth constitutes a large share of all household wealth. A common view in the public finance literature is that housing enjoys a tax favored status in most western economies, mainly because the return to owner housing, the imputed rent, usually goes untaxed while the return to business capital is taxed at a relatively high effective tax rate.[1] Related to this, several studies have assessed the welfare consequences of a tax reform that sets an equal tax burden on housing and business capital. Using quantitative dynamic general equilibrium models, Gahvari (1985), Skinner (1996), and Gervais (2002), among others, have shown that such a reform would lead to substantial efficiency gains.[2]

While these studies show that the current tax status of housing is likely to be highly distortionary, they do not aim to determine the *optimal* tax treatment of housing. Therefore, it is not clear that taxing the return to housing capital at the same rate as the return to business capital is always the best thing to do. In particular, because of the dual role of housing as both an asset and a consumption good, it seems plausible that the tax treatment of housing should depend on household preferences. In this paper, we consider housing taxation as part of an optimal tax problem and ask under what conditions it actually is optimal to tax the return to housing capital at the same rate as the return to business capital.

A related question that we are interested in is how the optimal tax treatment of business capital changes if housing capital cannot be taxed. Our motivation for this question is that most countries simply do not have a tax on the imputed rent and suggestions to introduce such a tax often face strong political opposition.[3] This may be considered as a constraint

[1] See Hendershott and White (2000) for an international comparison of housing's tax status.

[2] Other studies that also consider the efficiency and welfare effects of the tax favored status of housing include Gahvari (1984a), Slemrod (1982), Berkovec and Fullerton (1992), Hendershott and Won (1992), Poterba (1992), and Bye and Åvitsland (2003) and Eerola and Määttänen (2006). Turnovsky and Okuyama (1994) focus solely on capital accumulation. See also Englund (2003) for general discussion on housing taxation.

[3] Property taxes are common. However, since they are typically collected at the municipal level, their use may be limited by local tax competition.

that should be taken into account when designing tax reforms.

We follow the line of research represented by Judd (1985), Chamley (1986), Atkeson et al. (1999), and others. Formally, we analyze a Ramsey problem for a government that finances government expenditure by a set of flat rate taxes, including a tax on the imputed rental income from housing capital. The government is assumed to be able to commit to future tax policies. The solution to the Ramsey problem is a tax reform which is optimal given the initial state of the economy, individual optimization, and the available tax instruments. This approach also allows us to take the transitionary dynamics properly into account. This is important because the previous studies on housing taxation typically consider only steady state effects of tax reforms.[4]

We employ a version of the neoclassical growth model with a representative household that derives utility from non-housing consumption, housing, and leisure. The model captures the dual role of housing capital as an asset and a form of consumption, the intertemporal savings-consumption decision, and the general equilibrium effects of capital taxation we are interested in. In addition to introducing housing capital and a tax on it, we extend the most standard set-up by employing a nested constant elasticity of substitution (CES) utility function. To the best of our knowledge, we are the first to consider taxation of housing capital or durable goods as part of an optimal tax problem in a fully dynamic general equilibrium set-up.[5]

We first briefly discuss situations where the government has enough tax instruments to implement the first-best allocation. In such a set-up, housing and business capital should be treated equally independently of household preferences.

We then consider a standard second-best problem and show that there household preferences do matter for the optimal tax treatment of housing vis-à-vis business capital. Depending on the elasticities of substitution between non-housing consumption, housing, and leisure, it may be optimal to tax the imputed rent at a higher or lower rate than the return to business

[4]Exceptions are Skinner (1996) and Eerola and Määttänen (2006).

[5]Gahvari (1984b) studies the optimal taxation of housing capital in a partial equilibrium model. Cremer and Gahvari (1998) study the optimal taxation of housing in a static model with incomplete information, where the government may use differentiated housing taxes so as to separate between different consumer types.

capital. This is because housing taxation is used to alleviate the distorting effect of taxing labor.

We complement our analytical results with a numerical analysis. We show, first of all, that the optimal tax burden on housing capital is quite sensitive to the intratemporal elasticities of substitution.

We also show that the dynamics of the optimal tax rate on business capital change dramatically if housing capital cannot be taxed. In that case, the tax rate on the return to business capital does not feature the usual dynamics with very high tax rates in the first periods and a rapid convergence to the steady state level. Instead, the optimal business capital tax rate decreases very slowly and steadily towards the steady state level. Intuitively, if households can use housing capital as a tax free savings vehicle, taxing business capital at a high rate would induce a large reallocation from business to housing capital which would be undesirable. Related to this, we find that the welfare cost of not being able to tax housing is quite large even in a situation where the optimal long-run tax rate on housing is zero.

We proceed as follows. In the next section we describe the economy. In section 3, we present our analytical results. In section 4, we present and discuss our numerical results. We conclude in section 5. In the Appendix, we derive our analytical results.

2 Model

We consider a deterministic model with an infinitely lived representative household that derives utility from non-housing consumption, housing services, and leisure. The production side consists of a representative firm that employs business capital and labor to produce output goods which can be transformed into housing capital, business capital, and non-housing consumption. There is a government that finances public expenditures with flat-rate taxes.

We do not have residential land in the model. Hence, we focus on the tax treatment of housing capital, or residential structures, alone. The main reason is that it seems obvious that if constructible land is in fixed supply, the government should try to effectively confiscate all land rents. In practice, this can be (at least partly) achieved with municipal monopoly power on land use decisions. The question why we do not usually observe land confiscation is a very interesting one but goes beyond the scope of this paper.[6]

2.1 Firms

Every period t, a representative firm employs business capital, k_t, and labor, n_t, to produce output goods, y_t. The production function is

$$y_t = f(k_t, n_t). \tag{1}$$

Production function exhibits constant returns to scale. The firm's first-order conditions for profit maximization imply that the before-tax returns to business capital and labor are determined by their marginal productivities, that is,

$$\begin{aligned} r_t &= f_{k_t} - \delta_k & (2) \\ w_t &= f_{n_t}, & (3) \end{aligned}$$

where δ_k is the depreciation rate of business capital.[7] The output good may be costlessly converted into non-housing consumption good, business capital, and housing capital.

2.2 Government

The government finances each period amount g of public expenditures by taxing labor income at rate τ^n, the return to business capital income at rate τ^k, and the imputed rent at rate τ^h. The government budget need not be balanced on a period by period basis. Instead, the government faces the following budget constraint:

$$\tau_t^k r_t k_t + \tau_t^n n_t w_t + \tau_t^h r_t^h h_t + b_{t+1} - R_t b_t \geq g, \tag{4}$$

where b_t denotes a one-period bond maturing in period t and R_t is the gross rate of return on the bonds from period $t-1$ to period t. We assume that the return to government bonds is not taxed. This is innocuous for bond exchanges between the government and the household.

[6] See Aura and Davidoff (2006) for an interesting analysis about land taxation in a static spatial model and Davis and Heathcote (2005) for a calibrated business cycle model with housing and land.

[7] We denote $\frac{\partial}{\partial k_t} f(k_t, n_t) = f_{k_t}$ and similarly for other derivatives throughout the paper.

2.3 Household's problem

The household is endowed with one unit of time every period. The periodic utility function is $u(c, h, n)$, where c is non-housing consumption, h stock of housing capital, and n labor. The utility function is strictly increasing in non-housing consumption and housing capital and strictly decreasing in labor, strictly concave, and to satisfy the Inada conditions. The household has three savings vehicles: housing, business capital, and government bonds.

The maximization problem of the household in period 1 is

$$\max \sum_{t=1}^{\infty} \beta^{t-1} u\left(c_t, h_t, n_t\right) \tag{5}$$

subject to budget constraint

$$c_t + k_{t+1} + h_{t+1} + b_{t+1} = \left(1 + (1 - \tau_t^k) r_t\right) k_t + R_t b_t + R_t^h h_t + (1 - \tau_t^n) n_t w_t \tag{6}$$

where

$$R_t^h = 1 - \delta_h - \tau_t^h r_t^h.$$

In the budget constraint, r^h is the imputed rent (to be defined below) and δ_h is the depreciation rate of housing capital. The left hand side of the constraint includes expenditures on non-housing consumption, and investment or savings in housing, business capital and government bonds. The terms in the right hand side are after-tax income from business capital, owner-housing and labor.

It is clear that in this setting investment in business capital and savings in government bonds must have the same after-tax rate of return. This means that

$$R_t = 1 + (1 - \tau_t^k) r_t. \tag{7}$$

By recursively using the budget constraints to eliminate b_t terms and by taking into account the transversality condition, the periodic budget constraints can be merged into a single present-value budget constraint

$$\sum_{t=1}^{\infty} p_t [c_t + k_{t+1} + h_{t+1}] = \sum_{t=1}^{\infty} p_t \left[R_t^h h_t + \left(1 + (1 - \tau_t^k) r_t\right) k_t + (1 - \tau_t^n) n_t w_t\right] + R_1 b_1 \tag{8}$$

where p_1 and $p_t = \prod_{i=1}^{t-1} R_{i+1}^{-1}$ for $t > 1$.[8]

The first-order conditions characterizing individually optimal behavior may be written as

$$\begin{aligned} u_{c_t}(1-\tau_t^n)w_t + u_{n_t} &= 0 && (9) \\ u_{c_t} - \beta u_{c_{t+1}}\left(1+(1-\tau_t^k)r_t\right) &= 0 && (10) \\ \beta u_{h_{t+1}} - u_{c_t} + \beta u_{c_{t+1}} R_{t+1}^h &= 0. && (11) \end{aligned}$$

2.4 Imputed rent

We define the imputed rent as the user cost of housing net of depreciation and taxes. Hence, the imputed rent is simply

$$r_t^h = r_t. \tag{12}$$

The imputed rent can also be derived as the rental rate of housing, net of deprecation, that would prevail if rental markets existed in this economy. This hypothetical rental rate, denoted by $rent$, is determined by an arbitrage condition in the following manner. Investing one unit of output good in business capital in period $t-1$ returns $1+(1-\tau_t^k)r_t$ units in period t. One unit of output good buys one unit of housing capital. If we assume that the return to rental housing is taxed at the same rate as the return to business capital, and that landlords can deduct housing capital depreciation before paying the capital income tax, investing one unit of the output good in period $t-1$ in rental housing returns

$$rent_t - \tau_t^k\left(rent_t - \delta_h\right) + 1 - \delta_h \tag{13}$$

in period t. The last term in this expression is the value of the remaining housing capital in period t. The rental rate that makes households indifferent between investing in business capital and rental housing is

$$rent_t = r_t + \delta_h. \tag{14}$$

It is important to understand, however, that since we allow the tax rate on the imputed rent to be time varying, the allocations associated with optimal tax reforms do not depend on how exactly we define the tax base in housing taxation.

[8] For details on the formulation of the present-value budget constraint, see e.g. Ljungqvist and Sargent (2004, p. 482-483).

2.5 Equilibrium

For a given sequence of tax rates, a competitive equilibrium consists of individual policies and prices such that the individual policies solve the household's problem in (5) and (8), factor returns are given by equations in (2) and (3), the government budget constraint in (4) is satisfied with equality, and the aggregate resource constraint

$$c_t + k_{t+1} + h_{t+1} + g = f(k_t, n_t) + (1 - \delta_k)k_t + (1 - \delta_h)h_t \tag{15}$$

is satisfied for all t.

2.6 Note on aggregation and the tax treatment of mortgages

In the household problem above, we directly imposed the general equilibrium condition that the representative household owns the business capital stock. This conceals important issues related to way that households finance their housing and, in particular, the tax treatment of mortgage interests.

However, focusing on the representative household is less restrictive than it may seem at first glance. Under quite weak assumptions, the neoclassical growth model has the property that the dynamics of aggregate variables are independent of the underlying distribution of households over their asset holdings and labor productivities. In that case, we may assume that behind the representative agent there is a distribution of households with very different asset positions.

The key condition for this model economy to aggregate is that the after-tax interest rate is the same for all households. As for the utility function, it is sufficient to assume that it is homothetic.[9]

The after-tax interest rate being the same for all households requires in general that mortgage interest payments are tax deductible. To see this, it is useful to rewrite the periodic budget constraint so that it contains separately financial savings $a \geq 0$ and a mortgage $m \geq 0$. Assume also that fraction $\tau^m \leq 1$ of mortgage interest payments is tax deductible

[9] For a general analysis of aggregation in the neoclassical growth model, see e.g. Krusell and Ríos-Rull (1999) or Caselli and Ventura (2000). For a housing application, see Eerola and Määttänen (2006).

at the business capital tax rate. Then the household budget constraint defined in (6) can be rewritten as

$$c_t + a_{t+1} - m_{t+1} + h_{t+1} + b_{t+1} = y_t + (1 - \tau_t^n) n_t w_t \tag{16}$$

where

$$y_t = \left(1 + (1 - \tau_t^k) r_t\right) a_t - \left(1 + (1 - \tau_t^k \tau^m) r_t\right) m_t + R_t b_t + R_t^h h_t \tag{17}$$

is net worth of the household in period t. It is clear that if $\tau^k \neq 0$, the after-tax interest rate is the same for those having financial savings and for those having a mortgage only if $\tau^m = 1$, i.e. if mortgage interest payments are fully tax deductible. When this is the case, financial savings a and mortgage m can be combined into a single variable which in general equilibrium aggregates into the capital stock.

As discussed by Gervais (2002) and others, unless mortgage interest payments are fully tax deductible, the user cost of housing depends on whether it is financed with equity or debt. To see this, assume that a household with net worth y_t wishes to transfer a net worth of y_{t+1} to the following period and use (17) for period $t + 1$ to write

$$b_{t+1} = \frac{1}{R_{t+1}} \left[y_{t+1} - \left(1 + (1 - \tau_{t+1}^k) r_{t+1}\right) a_{t+1} + R_{t+1}^m m_{t+1} - R_{t+1}^h h_{t+1} \right], \tag{18}$$

where $R_t^m = 1 + (1 - \tau_t^k \tau^m) r_t$. Plugging the above expression for b_{t+1} into (16), taking into account the arbitrage condition in (7) and rearranging terms gives

$$c_t = Y_t - \frac{\tau_{t+1}^k (1 - \tau^m) r_{t+1}}{R_{t+1}} m_{t+1} - \frac{R_{t+1} - R_{t+1}^h}{R_{t+1}} h_{t+1} \tag{19}$$

where $Y_t = y_t + (1 - \tau_t^n) n_t w_t - \frac{y_{t+1}}{R_{t+1}}$. This equation shows how much current non-housing consumption the household can afford given next-period net worth, housing, and mortgage. With $\tau^m < 1$, a larger mortgage reduces current consumption. Housing is then more costly to households who finance it with a mortgage than to households who finance it with their own savings. With $\tau^m = 1$, the mortgage term drops out.

In short, assuming that mortgage interests are tax deductible and the utility function is homothetic, and taking into account that we are interested in aggregate efficiency alone (captured by the welfare of the representative agent), the optimal tax policy is independent of the initial distribution of households over their asset positions. Assuming that mortgage interest payments are tax deductible is natural because otherwise the tax system would be non-neutral between different ways of financing housing.

2.7 Specification of the preferences

For most of our analytical results, and obviously for all numerical results, we need to specify the utility function. We consider the following nested constant elasticity of substitution (CES) utility function.

The first CES aggregator, $\widehat{c}$, is defined over housing capital and leisure:

$$\widehat{c} = \widehat{c}(h, 1-n) = \begin{cases} \left(\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}\right)^{1/\gamma^h}, \text{ for } \gamma^h < 1, \gamma^h \neq 0 \\ h^{\theta^h}(1-n)^{1-\theta^h}, \text{ for } \gamma^h = 0, \end{cases}$$

where θ^h is the utility share of housing capital. The elasticity of substitution between housing capital and leisure is $\frac{1}{1-\gamma^h}$.

The second CES aggregator, $\widetilde{c}$, is defined over non-housing consumption and $\widehat{c}$:

$$\widetilde{c} = \widetilde{c}(c, \widehat{c}) = \begin{cases} \left(\theta^c c^{\gamma^c} + (1-\theta^c)\,\widehat{c}^{\gamma^c}\right)^{1/\gamma^c}, \text{ for } \gamma^c < 1, \gamma^c \neq 0 \\ \left(c^{\theta^c}\widehat{c}^{1-\theta^c}\right), \text{ for } \gamma^c = 0, \end{cases}$$

where θ^c is the utility share of non-housing consumption. The elasticity of substitution between non-housing consumption and $\widehat{c}$ is $\frac{1}{1-\gamma^c}$.

Finally, the periodic utility function is:

$$u(c, h, n) \equiv \widetilde{u}(\widetilde{c}(c, \widehat{c}(h, 1-n)) = \begin{cases} \frac{\widetilde{c}^{1-\sigma}}{1-\sigma}, \text{ for } \sigma > 0, \sigma \neq 1 \\ \log(\widetilde{c}), \text{ for } \sigma = 1, \end{cases}$$

where σ is the inverse of the intertemporal elasticity of substitution.

With $\gamma^c = \gamma^h = 0$, this utility function boils down to the commonly used Cobb-Douglas specification:

$$u(c, h, n) = \frac{[c^{\theta^c} h^{(1-\theta^c)\theta^h}(1-n)^{(1-\theta^c)(1-\theta^h)}]^{1-\sigma}}{1-\sigma}$$

The case with $\sigma = 1$ and $\gamma^c = \gamma^h = 0$ is the logarithmic utility function:

$$u(c, h, n) = \theta^c \log c + (1-\theta^c)\theta^h \log h + (1-\theta^c)(1-\theta^h)\log(1-n). \tag{20}$$

These preferences are equivalent to the set-up in Greenwood and Hercowitz (1991) who define a utility function over market consumption and 'home production' and a home production function over 'home capital' and time not allocated to market work.[10]

3 Optimal taxation

3.1 First-best

Before studying the second-best taxation, it is instructive to consider a first-best problem. As is well known, within the standard growth model the government can implement the first-best allocation if it has enough tax instruments and if the present value of government expenditures is small enough. In this subsection, we briefly extend some results from the previous literature to our set-up.

We note first that maximizing household utility subject to the aggregate resource constraint alone leads to the following first-order conditions:

$$u_{n_t} + u_{c_t} f_{n_t} = 0 \quad (21)$$

$$u_{c_t} - \beta u_{c_{t+1}} (1 + r_{t+1}) = 0 \quad (22)$$

$$\beta u_{h_{t+1}} - u_{c_t} + \beta u_{c_{t+1}} (1 - \delta_h) = 0. \quad (23)$$

These equations characterize the first-best allocation.

One way of implementing the first-best in the standard model without housing capital is to impose a constant strictly positive tax on consumption, a constant subsidy on labor income (i.e. $\tau^n < 0$), and a zero tax on the return to business capital (see Coleman, 2000). We next describe how this solution can be extended to our set-up. To do this, we need to introduce a consumption tax τ^c that applies to both non-housing consumption goods and housing capital (or residential construction).

[10] A more general formulation would allow for allocating time to 'leisure', 'home production' and 'market production'. For studies using this approach, see e.g. Gomme et al. (2001), Baxter and Jerman (1999) and McGrattan et al. (1997). The two approaches result in the same allocations under a logarithmic specification. For more discussion on this issue, see Greenwood et al. (1995).

With a consumption tax, the household budget constraint (8) becomes

$$\sum_{t=1}^{\infty} p_t[(1+\tau_t^c)\,c_t + k_{t+1} + (1+\tau_t^c)\,h_{t+1}] = \sum_{t=1}^{\infty} p_t\left[R_t^h h_t + \left(1+(1-\tau_t^k)r_t\right)k_t + (1-\tau_t^n)n_t w_t\right] + R_1 b_1 \tag{24}$$

where

$$R_t^h = (1-\delta_h)\,(1+\tau_t^c) - \tau_t^h r_t^h.$$

Consider then the following tax rates for all $t \geq 1$:

$$\tau_t^c = -\tau_t^n = \tau \geq 0 \tag{25}$$

$$\tau_{t+1}^k = \tau_{t+1}^h = 0 \tag{26}$$

Deriving the household first-order conditions using (24), inserting the tax policy in (25) and (26) into them, and comparing with (21)-(23) reveals that with this tax structure, the household first-order conditions are identical to those that characterize the first-best allocation. Hence, if the tax policy in (25) and (26) is feasible, the resulting competitive equilibrium will correspond to the first-best allocation. Intuitively, the government taxes initial assets with a consumption tax and eliminates the distorting effect on labor supply with a subsidy on labor.

For this solution to be feasible, it must generate enough tax revenue. Following Coleman (2000), it can be shown that this depends on the size of the initial aggregate capital stock relative to the present value of government expenditures.

Another way of reaching the first-best allocation is to allow for immediate expensing of investments as in Abel (2007). By denoting the investment credits for housing and business capital by ν^h and ν^k, respectively, the household budget constraint in (8) becomes

$$\sum_{t=1}^{\infty} p_t[c_t + k_{t+1} + h_{t+1}] = \sum_{t=1}^{\infty} p_t\left[R_t^h h_t + \nu_t^h I_t^h + \left(1+(1-\tau_t^k)r_t\right)k_t + \nu_t^k I_t^k + (1-\tau_t^n)n_t w_t\right] + R_1 b_1 \tag{27}$$

where

$$I_t^k = k_{t+1} - k_t \text{ and } I_t^h = h_{t+1} - (1-\delta_h)\,h_t.$$

Similar reasoning as above shows that the first-best can be implemented with constant positive tax rates on business and housing capital, a zero tax rate on labor, and investment credit satisfying $\nu^k = \tau^k$ and $\nu^h = \tau^h$. The investment credits together with positive capital tax rates allow the government to tax the initial capital stock without distorting the incentive to make new investments.

In should be noted that these solutions were derived without specifying the utility function. Hence, in the first-best tax system housing and business capital are always treated equally.

An important feature of these solutions is that they rely on taxing initial assets alone. They are therefore essentially capital levies and therefore arguably politically unacceptable. It is also likely that these tax systems do not generate enough tax revenue in countries with a relatively large government. These considerations motivate the analysis of a second-best tax problem.

3.2 Second-best

We now consider the second-best problem where the government does not have enough tax instruments to effectively confiscate the initial capital stocks. The objective of the government is to maximize household welfare by announcing in the beginning of period 1 a sequence of tax rates $\{\tau_t^n, \tau_{t+1}^k, \tau_{t+1}^h\}_{t=1}^{\infty}$. Note that we assume, following most of the related literature, that the government takes τ_1^k and τ_1^h as given. This rules out taxing past investments. We also define upper bounds $\overline{\tau}^k$ and $\overline{\tau}^h$ that the tax rates on the return to business capital and imputed rent may not exceed in any period $t \geq 2$.

Following the approach taken by Chamley (1986), Judd (1985), and others, we formulate the government's problem so that it directly chooses allocation $\{c_t, n_t, k_{t+1}, h_{t+1}\}_{t=1}^{\infty}$.[11] Before writing down the government objective, let us discuss the constraints to be imposed on the government's choices.

Rewriting the budget constraint of the household in (8) by using the first-order conditions

[11] As shown in Lansing (1999) and discussed in Krusell (2002), in some cases, this approach leads to allocations that cannot be decentralised. This happens in the absence of anticipation effects, that is, when future tax rates do not affect the current decisions of the private sector. In our setting this kind of anticipation effects are always present even under logarithmic utility.

of the household gives

$$\sum_{t=1}^{\infty}\beta^{t-1}\left(u_{c_t}c_t+u_{n_t}n_t+\beta u_{h_{t+1}}h_{t+1}\right)=u_{c_1}A. \tag{28}$$

where $A=\left(1+\left(1-\tau_1^k\right)r_1\right)k_1+R_1^h h_1+R_1 b_1$. This is the so-called implementability constraint. It states that the allocation chosen by the government must be compatible with individual optimization. Notice that unlike business capital, housing capital enters the implementability constraint. This is because housing capital is in the utility function.

The upper bounds on the two capital tax rates are imposed as follows: First, given (10), requirement $\tau_t^k\leq\overline{\tau}^k$ means that the government is constrained to choose allocations that satisfy

$$u_{c_{t-1}}\geq\beta u_{c_t}\left(1+(1-\overline{\tau}^k)r_t\right). \tag{29}$$

Second, given (11), requirement $\tau_t^h\leq\overline{\tau}^h$ translates into condition

$$u_{c_{t-1}}\geq\beta u_{h_t}+\beta u_{c_t}\left(1-\delta_h-\overline{\tau}^h r_t^h\right). \tag{30}$$

The Lagrangian for the government may be written as:

$$\begin{aligned}\mathcal{L}\ =\ &\sum_{t=1}^{\infty}\beta^{t-1}u(c_t,h_t,n_t)\\&+\lambda\left[\sum_{t=1}^{\infty}\beta^{t-1}\left(u_{c_t}c_t+u_{n_t}n_t+\beta u_{h_{t+1}}h_{t+1}\right)-u_{c_1}A\right]\\&+\sum_{t=1}^{\infty}\beta^{t-1}\mu_t\left[f(k_t,n_t)+(1-\delta_k)k_t+(1-\delta_h)h_t-c_t-k_{t+1}-h_{t+1}-g\right]\\&+\sum_{t=1}^{\infty}\beta^{t-1}\omega_t\left[u_{c_t}-\beta u_{c_{t+1}}\left(1+(1-\overline{\tau}^k)r_{t+1}\right)\right]\\&+\sum_{t=1}^{\infty}\beta^{t-1}\psi_t\left[u_{c_t}-\beta u_{h_{t+1}}-\beta u_{c_{t+1}}\left(1-\delta_h-\overline{\tau}^h r_{t+1}^h\right)\right].\end{aligned} \tag{31}$$

The first constraint is the implementability constraint. The second set of constraints contains an aggregate resource constraint for each period. The third and fourth sets of constraints are the restrictions on the tax rates.

For $t > 1$, the first-order conditions for the government are:[12]

$$n_t \ : \ W_{n_t} + \mu_t f_{n_t} + B_{t-1} u_{cn_t} - \omega_{t-1}\left(1-\overline{\tau}^k\right) u_{c_t} f_{kn_t} - \psi_{t-1}\left(u_{hn_t} - u_{c_t}\overline{\tau}^h f_{n_t}\right) = 0 \quad (32)$$

$$c_t \ : \ W_{c_t} - \mu_t + B_{t-1} u_{cc_t} - \psi_{t-1} u_{hc_t} = 0 \quad (33)$$

$$k_{t+1} \ : \ -\mu_t + \beta\mu_{t+1}\left(1+r_{t+1}\right) - \omega_t\left(1-\overline{\tau}^k\right) u_{c_{t+1}} f_{kk_{t+1}} + \psi_t u_{c_{t+1}} \overline{\tau}^h f_{k_{t+1}} = 0 \quad (34)$$

$$h_{t+1} \ : \ \beta W_{h_{t+1}} - \mu_t + \beta\mu_{t+1}\left(1-\delta_h\right) + \beta B_t u_{ch_{t+1}} - \psi_t \beta u_{hh_{t+1}} = 0 \quad (35)$$

where

$$\begin{aligned} B_t &= \omega_{t+1} - \omega_t\left(1+(1-\overline{\tau}^k)r_{t+1}\right) + \psi_{t+1} - \psi_t\left(1-\delta_h-\overline{\tau}^h r_{t+1}^h\right) \\ W_{n_t} &= u_{n_t} + \lambda(u_{hn_t}h_t + u_{cn_t}c_t + u_{nn_t}n_t + u_{n_t}) \\ W_{c_t} &= u_{c_t} + \lambda(u_{hc_t}h_t + u_{cc_t}c_t + u_{c_t} + u_{nc_t}n_t) \\ W_{h_t} &= u_{h_t} + \lambda(u_{hh_t}h_t + u_{h_t} + u_{ch_t}c_t + u_{nh_t}n_t). \end{aligned}$$

We also have the following Kuhn-Tucker conditions for all $t \geq 1$:

$$\psi_t\left[u_{c_t} - \beta u_{c_{t+1}}\left(1-\delta_h-\overline{\tau}^h r_{t+1}^h\right) - \beta u_{h_{t+1}}\right] = 0,\ \psi_t \geq 0, \quad (36)$$
$$\text{and } u_{c_t} - \beta u_{c_{t+1}}\left(1-\delta_h-\overline{\tau}^h r_{t+1}^h\right) \geq \beta u_{h_{t+1}}.$$

$$\omega_t\left[u_{c_t} - \beta u_{c_{t+1}}\left(1+(1-\overline{\tau}^k)r_{t+1}\right)\right] = 0,\ \omega_t \geq 0, \quad (37)$$
$$\text{and } u_{c_t} - \beta u_{c_{t+1}}\left(1+(1-\overline{\tau}^k)r_{t+1}\right) \geq 0.$$

The optimality conditions (32)-(37), the aggregate resource constraint (15), and the implementability constraint (28) determine the allocation, $\{c_t, n_t, k_{t+1}, h_{t+1}\}_{t=1}^{\infty}$, as well as the multipliers λ and $\{\mu_t, \omega_t, \psi_t\}_{t=1}^{\infty}$. After an optimal allocation has been found, prices, $\{r_t, w_t\}_{t=1}^{\infty}$, are determined from equations (2) and (3). Finally, the tax rates on labor, the return to business capital, and the imputed rental income are solved from equations (9), (10), and (11), respectively.

By combining the steady state versions of the government first-order condition (34) and the household first-order condition (10), it is straightforward to show that in the long run the tax rate on the return to business capital should be zero. This is the standard Chamley-Judd result.

[12]The first-order conditions for c_1 and n_1 (not shown) look somewhat different because they affect the righ-hand side of (28).

Our interest is in comparing the optimal tax treatment of housing capital to that of business capital not just in the steady state but also during the transition. In what follows, we do this with two analytical results. When deriving these results, we ignore the upper bounds on the capital tax rates, $\overline{\tau}^k$ and $\overline{\tau}^h$, which are typically binding only during the first periods after the optimal tax reform is announced. Hence, strictly speaking, the following results hold starting from the first period when neither of these upper bounds is binding.

By using the government's first-order conditions (33)-(35) and assuming that $\omega = \psi = 0$, we obtain

$$\frac{W_{h_{t+1}}}{W_{c_{t+1}}} - (\delta_h + r_{t+1}) = 0. \tag{38}$$

By using household's first-order conditions (10) and (11), we obtain

$$\frac{u_{h_{t+1}}}{u_{c_{t+1}}} = \left(1 - \tau^k_{t+1}\right) r_{t+1} + \delta_h + \tau^h_{t+1} r^h_{t+1}. \tag{39}$$

Our strategy is to combine (38) and (39) in a manner that will allow us to determine the relationship of different tax rates. In order to do that, we consider the utility function defined in subsection 2.7. Details of the analysis are in the Appendix.

We obtain the following result:

Result 1 *If $\gamma^c = \gamma^h$, then $\tau^k_t = \tau^h_t$ in all periods.*

In words, the result shows that for a class of utility functions that includes Cobb-Douglas preferences, the second-best optimum can be achieved with a tax structure where the tax rate on the imputed rent equals the tax rate on the return to business capital.

However, whenever $\gamma^c \neq \gamma^h$, the result that housing and business capital should be taxed at the same rate breaks down. We have the following result:

Result 2 *If $\gamma^c \neq \gamma^h$, then $\tau^k_t \neq \tau^h_t$ in all periods. In addition,*

i) If $\sigma = 1, \gamma^h > \gamma^c$, and $\gamma^h > 0$, then $\tau^h_t < \tau^k_t$ in all periods.

ii) If $\sigma = 1$, $\gamma^h < \gamma^c$, and $\gamma^h \leq 0$ but close enough to zero, then $\tau^h_t > \tau^k_t$ in all periods.

Parts i) and ii) show that the imputed rent should be taxed at a lower (higher) rate than the return to business capital if γ^h is larger (smaller) than γ^c, at least when $\sigma = 1$ and γ^h is not too small.

To interpret this result, note first that the elasticity of substitution between housing and leisure increases with γ^h. If the elasticity of substitution between housing and leisure is high, a small increase in the tax burden on housing leads the household to demand a lot more leisure (which cannot be taxed). In such a situation, the tax burden on housing should be low. However, the relevant elasticity depends also on γ^c. For a fixed γ^h, a higher value of γ^c means that the household is more willing to substitute both housing and leisure for non-housing consumption. In other words, if γ^c is high, a small increase in the tax burden on housing leads the household to demand a lot less leisure. Hence, the tax burden on housing should decrease with γ^h and increase with γ^c.

More generally, this reflects the well-known result that goods that are sufficiently strong substitutes for leisure should be taxed at a lower rate than other goods (see e.g. Christiansen, 1984) in order to alleviate the distortionary effect of labor taxation. However, we have not seen this result been derived before in a fully dynamic set-up. Here, alleviating the distortionary effect of labor taxation comes at the cost of distorting a dynamic investment decision.

Technically, the reason why the optimal tax treatment of housing capital may differ from that of business capital is that unlike business capital, housing capital enters the utility function. Hence, Result 2 stems from the consumption role of housing. The fact that we have modelled housing as owner housing is not crucial. What matters is the price of housing vis-à-vis the price of non-housing consumption and leisure. We could equivalently introduce a firm that borrows from the households, invests in housing capital and rents it to the households. Housing could then be taxed at the firm level and the tax system would determine the cost of housing via a zero-profit condition.

4 Numerical analysis

In our view, the most important questions that our analytical results leave open are the following: First, what are the transitional dynamics of the optimal tax rates in the second-best solution? Second, how do the dynamics change if housing cannot be taxed at all? Third, how sensitive is the optimal long run tax rate on the imputed rent with respect to changes in the CES preference parameters? Fourth, what is the welfare cost of not taxing housing? In this section, we present numerical results to answer these questions.[13]

4.1 Calibration

We take the model period to be one year and calibrate the model to the US economy. We assume that the production function is Cobb-Douglas with capital share α. Greenwood et al. (1995) have estimated the share of business capital in the production function when total capital stock is disaggregated into housing and business capital. Based on their estimate, we set $\alpha = 0.29$. We set the depreciation rates of business and housing capital equal to the average depreciation rates of nonresidential and residential fixed assets in 2000-08 National Income and Product Accounts (NIPA). This implies $\delta_k = 0.061$ and $\delta_h = 0.015$.[14]

In the baseline calibration, we set $\gamma^c = \gamma^h = 0$. This is consistent with the fact that housing expenditure share and average hours worked have been fairly constant over the last decades in the US (Kydland, 1995). However, studies using micro data often find different results.[15] We will therefore conduct a sensitivity analysis with respect to both of these intratemporal elasticity parameters. In addition, we set $\sigma = 1$ so that we have the logarithmic utility function in (20).

Based on Carey and Rabesona (Table 7.2., 2004), we set $\tau_0^n = 0.234$. We further set the initial tax rate on the imputed rent to zero, i.e. $\tau_0^h = 0$. We choose the preference parameters β, θ^c, θ^h, government consumption, g, and the tax rate on the return to business capital, τ_0^k, so as to match the following targets: 1) Total capital to total output ratio $(k + h)/y = 2.96$, where $y = k^\alpha n^{1-\alpha} + \left(r^h + \delta_h\right) h$. 2) Housing capital to business capital ratio $h/k = 0.76$. 3)

[13] We find the solution to the Ramsey problem by solving the system of non-linear equations formed by the government first-order conditions, the implementability constraint, the aggregate resource constraints, and the Kuhn-Tucker constraints using the broydn's algorithm. The Matlab programs are available from the authors upon request.

[14] These depreciation rates equal the ratios of real depreciation to the real net stocks of non-residential and residential fixed assets (private and government).

[15] We are not aware of a study that would have estimated the same preference structure that we have here with micro data. However, there are a number of studies that use micro data to estimate the elasticity of substitution between housing and non-housing consumption. These studies tend to find that elasticity to be quite small. See for instance Siegel (2005) and Flavin and Nagazawa (2008).

Government consumption to total output ratio $g/y = 0.19$. 4) Labor supply $n = 0.33$. 5) Government budget is balanced and there is no government debt.

The first three targets are based on NIPA. The first is the ratio of fixed assets (private and government) to GDP and the second is the ratio of residential fixed assets to non-residential fixed assets. The third target is the ratio of government consumption expenditures and gross investment to GDP.[16] The fourth target means that the representative household uses one third of its time endowment working. All the parameter values are collected in Table 1.

Table 1: Baseline calibration.

	Parameter	Value	Definition
Preferences	β	0.942	Discount factor
	θ^c	0.296	Consumption share parameter
	θ^h	0.128	Housing share parameter
	γ^c	0	CES parameter
	γ^h	0	CES parameter
	σ	1	Intertemporal elasticity parameter
Technology	α	0.290	Business capital share
	δ_k	0.061	Depreciation rate of business capital
	δ_h	0.015	Depreciation rate of housing capital
Tax system	τ_0^n	0.234	Initial labor tax
	τ_0^h	0	Initial tax rate on the imputed rent
	τ_0^k	0.308	Initial tax rate on the return to business capital
	g	0.096	Government expenditures

4.2 Transitionary dynamics

In this subsection, we display the optimal dynamic tax reform in the baseline calibration. We also illustrate how it changes if the imputed rent cannot be taxed. In that case, we set $\overline{\tau}^h = 0$. Figure 1 shows the paths of the optimal tax rates. Period 0 corresponds to the initial steady state. The tax rate paths in the left hand side of the figure correspond to the case where the imputed rent can be taxed. Here we have set the upper bounds for the two capital

[16] All these targets are the average ratios for 2000-08.

tax rates at 1, that is, $\overline{\tau}^k = \overline{\tau}^h = 1$.[17] The tax rate paths in the right hand side correspond to the case where the imputed rent cannot be taxed.

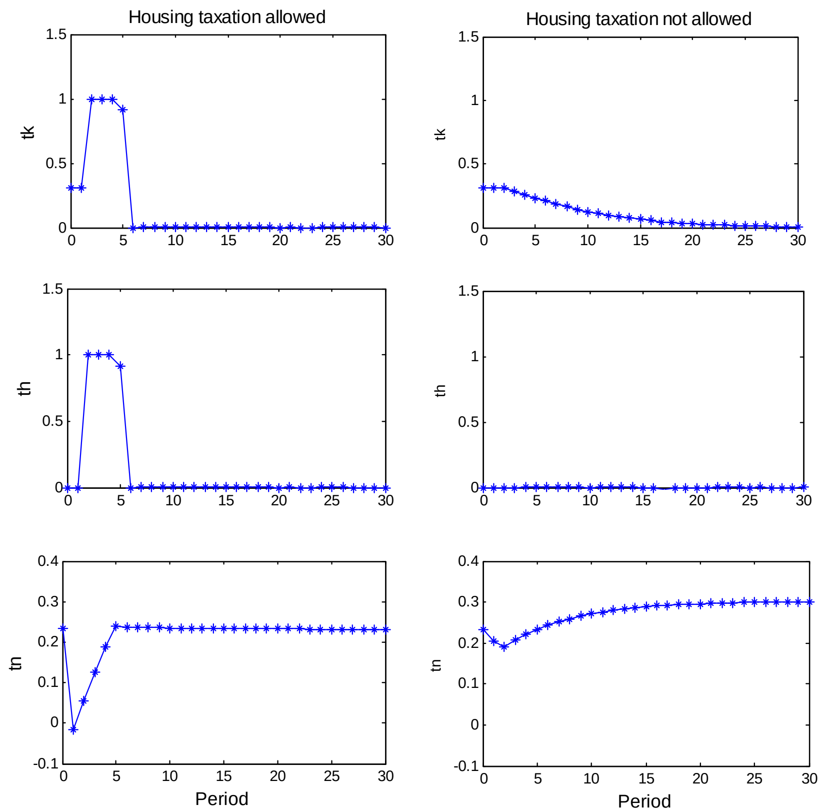

Figure 1: Optimal tax rates with (left) and without (right) the possibility to tax the imputed rent.

Consider first the case where the imputed rent can be taxed. From the analytical results we know that the tax rates on the return business capital and on the imputed rent should be equal in all periods. The figure shows that the upper bounds imposed on these tax rates are binding during the first three periods. After that, both tax rates converge to zero in two periods.

Interestingly, if the imputed rent cannot be taxed, the dynamics of the optimal tax rate on the return to business capital are very different in two respects: First, the tax rate now starts to diminish from period 2 onwards and the upper bound $\tau^k \leq 1$ is never binding.

[17]This is a natural upper bound for τ^k since the after-tax return to business capital would become negative with $\tau^k > 1$. In such a case, investors would refuse to hold any business capital. However, there is no such natural upper bound for τ^h. This is because households should always be willing to hold some housing capital as long as the marginal utility of housing goes to infinity when housing goes to zero.

Second, the tax rate on the return to business capital converges to zero very slowly. In fact, it appears to converge to zero only asymptotically. In other words, if the tax burden on housing cannot be adjusted, the tax rate on the return to business capital does not feature the usual dynamics with very high tax rates in the first periods and a rapid convergence to the new steady state tax rate. Intuitively, if households can use housing capital as a tax free savings vehicle, taxing business capital stock at a very high rate becomes undesirable because it would induce a large reallocation from business to housing capital.

4.3 Elasticities and the optimal long-run tax system

We know from result 2 that the optimal tax rate on the imputed rent differs from the optimal tax rate on the return to business capital whenever the intratemporal elasticity parameters γ^c and γ^h are not equal. We now illustrate the quantitative importance of this result by reporting the optimal long run tax rate on the imputed rent with different values for these two parameters. We consider values of -1 and 1/3 which correspond to elasticities of substitution equal to 0.5 and 1.5, respectively. When changing these parameters, we recalibrate parameters β, θ^c, θ^h, g, and τ_0^k so as to match the same targets as in the baseline calibration.[18]

In table 2, we display the optimal long run tax rates on the imputed rent and labor. The optimal long run tax rate on the return to business capital is always zero.

The optimal tax rate on the imputed rent varies with the elasticity parameters. In the table, the tax rate ranges from -0.24 to 0.57. The extreme cases are those where the difference between γ^c and γ^h is the largest. Hence, the optimal tax burden on housing is quite sensitive to household preferences.

Table 2: Long run tax rates.

[18] In fact, β remains constant when we vary the elasticity parameters.

Calibration	τ^h	τ^n
$\gamma^c = 0, \gamma^h = -1$	0.33	0.20
$\gamma^c = 0, \gamma^h = 0$	0.00	0.23
$\gamma^c = 0, \gamma^h = 1/3$	-0.11	0.24
$\gamma^c = -1, \gamma^h = 0$	-0.18	0.26
$\gamma^c = 0, \gamma^h = 0$	0.00	0.23
$\gamma^c = 1/3, \gamma^h = 0$	0.16	0.21
$\gamma^c = -1, \gamma^h = 1/3$	-0.24	0.27
$\gamma^c = 1/3, \gamma^h = -1$	0.57	0.17

4.4 Welfare effects

Finally, we consider the welfare effects of optimal tax reforms. Our welfare measure is the 'equivalent consumption variation'. It tells how much non-housing consumption should be increased in the initial steady state (keeping housing and leisure fixed) so as to make the household indifferent between the status quo and the equilibrium associated with a tax reform. We compare the welfare gains of optimal tax reforms with and without the possibility to tax the imputed rent. The difference between these two welfare gains gives us a measure of the welfare cost associated with the inability to tax housing. We compute two welfare effects: An overall welfare effect that takes the transition periods into account and a steady state welfare effect that is based on comparing utility in the initial steady state to the utility in the new steady state.

With the baseline calibration, the welfare gain from an optimal dynamic tax reform is 1.8%. The steady state effect is 8.7%. The reason why the steady state welfare gain is much larger than the overall welfare gain is that aggregate capital stock increases with the optimal tax reform. Following the tax reform, the savings rate initially increases.

The overall welfare gain of the optimal tax reform falls to 0.7% and the steady state effect to 4.8% if the tax rate on the imputed rent is constrained to be zero. In other words, the welfare gain falls by about 50% if the imputed rent cannot be taxed. In this sense, the cost of not being able to tax the imputed rent is very large. This is perhaps surprising given that the initial tax burden on housing (with $\tau_0^h = 0$) is the same as the optimal tax burden in the new steady state. The reason for the large reduction in the welfare gain is imposing high tax rates on the return to business capital during the first periods of the reform becomes very distorting when households can use housing capital as a tax free savings vehicle.

5 Conclusions

We have considered the optimal tax treatment of housing capital within a version of the neoclassical growth model. In the first best, the tax treatment of housing and business capital should be the same. For a class of utility functions that includes the standard Cobb-Douglas function, also the second-best optimum calls for taxing the imputed rent at the same rate as the return to business capital. More generally, however, the optimal tax burden on housing depends on the elasticities of substitution between housing, non-housing consumption, and leisure.

Our numerical results suggest that the optimal tax treatment of housing capital is indeed quite sensitive to household preferences. This means that the optimal tax rate on the imputed rent may depart substantially from the optimal tax rate on the return to business capital. We also found that the dynamics of optimal capital tax reforms are very different if the tax burden on housing cannot be adjusted.

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Appendix

In this appendix, we derive Results 1 and 2. Throughout we assume that the upper bounds for the tax rates on the imputed rent and the return to business capital are not binding, that is $\omega = \psi = 0$. For convenience, we drop time indices.

We first prove the following lemma.

Lemma 1 *If* $\gamma^c = \gamma^h$, *then*

$$\frac{W_h}{W_c} = \frac{u_h}{u_c}.$$

where

$$W_c = u_c + \lambda(u_{hc}h + u_{cc}c + u_c + u_{nc}n) \tag{A1}$$

$$W_h = u_h + \lambda(u_{hh}h + u_h + u_{ch}c + u_{nh}n). \tag{A2}$$

Proof. We have to consider different cases depending on the parameter values in the utility function. Note first that for $\sigma \neq 1$, $\gamma^c = \gamma^h \neq 0$, the utility function we employ may be written as

$$\begin{aligned} u(c,h,n) &= \frac{\left[\theta^c c^{\gamma^c} + (1-\theta^c)\left(\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}\right)^{\gamma^c/\gamma^h}\right]^{(1-\sigma)/\gamma^c}}{1-\sigma} \\ &\equiv \frac{S\left(c,h,n\right)^{\frac{1-\sigma}{\gamma^c}}}{1-\sigma} \end{aligned}$$

where $S\left(c,h,n\right) = \theta^c c^{\gamma^c} + (1-\theta^c)\left(\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}\right)^{\gamma^c/\gamma^h}$. The partial derivatives of the utility function can then be written using the partial derivatives of S as follows:

$$u_i = \frac{1}{\gamma^c} S^{\varphi} S_i \text{ for } i = c,h,n \tag{A3}$$

$$\begin{aligned} u_{ij} &= \frac{1}{\gamma^c}\varphi S^{\varphi-1} S_i S_j + \frac{1}{\gamma^c} S^{\varphi} S_{ij} = u_i\left(\varphi\frac{S_j}{S} + \frac{S_{ij}}{S_i}\right) \\ &= u_j\left(\varphi\frac{S_i}{S} + \frac{S_{ij}}{S_j}\right) \text{ for } i,j = c,h,n \end{aligned} \tag{A4}$$

where $\varphi = \frac{1-\sigma}{\gamma^c} - 1$. The partial derivatives of S are

$$\begin{aligned} S_c &= \gamma^c \theta^c c^{\gamma^c - 1} \\ S_{cc} &= S_c\left(\gamma^c - 1\right)c^{-1} \\ S_h &= \gamma^c(1-\theta^c)D^{\gamma^c/\gamma^h - 1}\theta^h h^{\gamma^h - 1} \\ S_{hh} &= S_h\left[\left(\gamma^c - \gamma^h\right)D^{-1}\theta^h h^{\gamma^h - 1} + \left(\gamma^h - 1\right)h^{-1}\right] \\ S_n &= -\gamma^c(1-\theta^c)D^{\gamma^c/\gamma^h - 1}\left(1-\theta^h\right)(1-n)^{\gamma^h - 1} \\ S_{ch} &= S_{cn} = 0 \\ S_{hn} &= -S_h D^{-1}\left(\gamma^c - \gamma^h\right)\left(1-\theta^h\right)(1-n)^{\gamma^h - 1} \end{aligned}$$

where $D = \theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}$. Plugging the partial derivatives of u into (A1) and (A2) and rearranging gives:

$$\begin{aligned} W_c &= u_c + \lambda u_c \left(\frac{\varphi F}{S} + 1 + \frac{S_{cc}c}{S_c} + \frac{S_{cn}n}{S_c} + \frac{S_{ch}h}{S_c} \right) \\ W_h &= u_h + \lambda u_h \left(\frac{\varphi F}{S} + 1 + \frac{S_{hh}h}{S_h} + \frac{S_{hn}n}{S_h} + \frac{S_{ch}c}{S_h} \right) \end{aligned}$$

where $F = S_c c + S_n n + S_h h$. Plugging the partial derivatives of S into the above expressions allows us to write:

$$W_c = u_c \left[1 + \lambda \left(\frac{\varphi F}{S} + \gamma^c \right) \right] \tag{A5}$$

$$W_h = u_h \left[1 + \lambda \left(\frac{\varphi F}{S} + \gamma^h + \left(\gamma^c - \gamma^h\right) \frac{\theta^h h^{\gamma^h} - \left(1-\theta^h\right)\left(1-n\right)^{\gamma^h - 1} n}{\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}} \right) \right] \tag{A6}$$

From equation (A6) it follows that if $\gamma^c = \gamma^h$,

$$W_h = u_h \left[1 + \lambda \left(\frac{\varphi F}{S} + \gamma^h \right) \right].$$

Therefore, if $\gamma^c = \gamma^h$, (A5) and (A6) imply that

$$\frac{W_h}{W_c} = \frac{u_h}{u_c}.$$

Hence, we have proved Lemma 1 for the case where $\sigma \neq 1$ and $\gamma^c = \gamma^h \neq 0$. When $\sigma = 1$ and $\gamma^c = \gamma^h \neq 0$,

$$u(c, h, n) = \frac{1}{\gamma^c} \log S$$

It is clear that the above result applies in this case, because equations (A3)-(A4) and the analysis thereafter remain valid. Consider then the case where $\gamma^c = \gamma^h = 0$ and $\sigma = 1$. The utility function is then

$$u(c, h, n) = \theta^c \log c + (1-\theta^c)\theta^h \log h + (1-\theta^c)(1-\theta^h) \log(1-n).$$

Plugging the partial derivatives of u into (A1) and (A2) leads to

$$W_c = u_c \text{ and } W_h = u_h$$

and therefore Lemma 1 applies. Finally, consider the case where $\sigma \neq 1$ and $\gamma^c = \gamma^h = 0$. In this case

$$u(c,h,n) = \frac{\left[c^{\theta^c} h^{\theta^h(1-\theta^c)}(1-n)^{(1-\theta^h)(1-\theta^c)}\right]^{1-\sigma}}{1-\sigma} \equiv \frac{\widetilde{S}\,(c,h,n)^{1-\sigma}}{1-\sigma}$$

where $\widetilde{S} = c^{\theta^c} h^{\theta^h(1-\theta^c)}(1-n)^{(1-\theta^h)(1-\theta^c)}$. Then

$$\begin{aligned}
u_i &= \widetilde{S}^{-\sigma}\widetilde{S}_i \text{ for } i = c,h,n \\
u_{ij} &= -\sigma\widetilde{S}^{-\sigma-1}\widetilde{S}_i\widetilde{S}_j + \widetilde{S}^{-\sigma}\widetilde{S}_{ij} = u_i\left[-\sigma\widetilde{S}^{-1}\widetilde{S}_j + \frac{\widetilde{S}_{ij}}{\widetilde{S}_i}\right] \text{ for } i,j = c,h,n
\end{aligned}$$

Proceeding as above we can write

$$\begin{aligned}
\widetilde{S}_c &= \theta^c c^{-1}\widetilde{S} \\
\widetilde{S}_h &= \theta^h(1-\theta^c)h^{-1}\widetilde{S} \\
\widetilde{S}_n &= -(1-\theta^h)(1-\theta^c)\,(1-n)^{-1}\,\widetilde{S} \\
\widetilde{S}_{cc} &= \theta^c c^{-1}\widetilde{S}_c - c^{-1}\widetilde{S}_c \\
\widetilde{S}_{hh} &= \theta^h(1-\theta^c)h^{-1}\widetilde{S}_h - h^{-1}\widetilde{S}_h \\
\widetilde{S}_{ch} &= \theta^h(1-\theta^c)h^{-1}\widetilde{S}_c = \theta^c c^{-1}\widetilde{S}_h \\
\widetilde{S}_{cn} &= -(1-\theta^h)(1-\theta^c)\,(1-n)^{-1}\,\widetilde{S}_c \\
\widetilde{S}_{hn} &= -(1-\theta^h)(1-\theta^c)\,(1-n)^{-1}\,\widetilde{S}_h
\end{aligned}$$

Plugging these expressions into (A1) and (A2) we have

$$\begin{aligned}
W_c &= u_c + \lambda u_c\left(\frac{-\sigma\widetilde{F}}{\widetilde{S}} + \theta^c + \theta^h(1-\theta^c) - (1-\theta^h)(1-\theta^c)\,(1-n)^{-1}\,n\right) \text{ and} \\
W_h &= u_h + \lambda u_h\left(\frac{-\sigma\widetilde{F}}{\widetilde{S}} + \theta^c + \theta^h(1-\theta^c) - (1-\theta^h)(1-\theta^c)\,(1-n)^{-1}\,n\right).
\end{aligned}$$

where $\widetilde{F} = \widetilde{S}_c c + \widetilde{S}_n n + \widetilde{S}_h h$. Again, $\frac{W_h}{W_c} = \frac{u_h}{u_c}$. We have now proved Lemma 1. ∎

Proof of Result 1. Assume now that $\gamma^c = \gamma^h$. By using Lemma 1 we can write equation (38) as

$$\frac{u_h}{u_c} = \delta_h + r.$$

Inserting this expression into (39) gives

$$\begin{aligned}\delta_h + r &= \left[1 - \left(1-\tau^k\right) r - (1-\delta_h)\right] + \tau^h r \\ &\Leftrightarrow \\ \delta_h + r &= \left(1-\tau^k\right) r + \delta_h + \tau^h r\end{aligned}$$

This proves Result 1. ■

In order to prove Result 2 we first prove the following lemma:

Lemma 2. If $\gamma^c \neq \gamma^h$, then

$$\frac{W_h}{W_c} = \frac{u_h}{u_c}\frac{G}{C} \tag{A7}$$

where

$$G = 1 + \lambda S_{\cdot}^{-1}\left(\varphi F + S\gamma^h + S\left(\gamma^c - \gamma^h\right)\frac{\theta^h h^{\gamma^h} - \left(1-\theta^h\right)(1-n)^{\gamma^h - 1} n}{\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}}\right)$$

and

$$C = 1 + \lambda S_{\cdot}^{-1}\left(\varphi F + S\gamma^c\right).$$

Because $\gamma^c \neq \gamma^h$, it follows that

$$\frac{G}{C} \neq 1 \tag{A8}$$

In addition, when $\sigma = 1$,

$$\begin{aligned} G &> 1 \text{ if } \gamma^h > \gamma^c \text{ and } \gamma^h \geq 0 \\ 0 &< G < 1 \text{ if } \gamma^h < \gamma^c,\ \gamma^h \leq 0, \text{ and } \gamma^h \text{ is not too small.}\end{aligned}$$

Proof. Equation (A7) follows directly from equations (A5) and (A6). We use the expressions for partial derivatives for u and S from Lemma 1 in order to write

$$\begin{aligned} \frac{G}{C} &= 1 \Leftrightarrow \\ S\gamma^h + \varphi F + S\left(\gamma^c - \gamma^h\right)\frac{\theta^h h^{\gamma^h} - \left(1-\theta^h\right)(1-n)^{\gamma^h-1} n}{\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}} &= S\gamma^c + \varphi F \Leftrightarrow \\ S\left(\gamma^h - \gamma^c\right) + \varphi F + S\left(\gamma^c - \gamma^h\right)\frac{\theta^h h^{\gamma^h} - \left(1-\theta^h\right)(1-n)^{\gamma^h-1} n}{\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}} &= 0 \Leftrightarrow \\ \left(\gamma^h - \gamma^c\right)\left(\theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}\right)^{-1}(1-\theta^h)(1-n)^{\gamma^h}\left(1 + \frac{n}{1-n}\right) &= 0\end{aligned}$$

Clearly, the above equation is only satisfied when $\gamma^h = \gamma^c$. Hence, $\gamma^c \neq \gamma^h$ implies that $\frac{G}{C} \neq 1$.

In order to determine the sign of G note first that if $\sigma = 1$, then $\varphi = -1$. Therefore we can write G as

$$G = 1 + \frac{\lambda}{SD}\left[DS\gamma^h - DF + S\left(\gamma^c - \gamma^h\right)\left(\theta^h h^{\gamma^h} - \left(1-\theta^h\right)(1-n)^{\gamma^h-1} n\right)\right]$$

where $D = \theta^h h^{\gamma^h} + (1-\theta^h)(1-n)^{\gamma^h}$. We then proceed by simplifying the expression inside the brackets. We first write F as

$$\begin{aligned}
F &= S_c c + S_n n + S_h h \\
&= \gamma^c \theta^c c^{\gamma^c - 1} c - \gamma^c (1-\theta^c) D^{\gamma^c/\gamma^h - 1}\left(1-\theta^h\right)(1-n)^{\gamma^h-1} n + \gamma^c(1-\theta^c) D^{\gamma^c/\gamma^h-1} \theta^h h^{\gamma^h-1} h \\
&= \gamma^c\left[\theta^c c^{\gamma^c} + (1-\theta^c) D^{\gamma^c/\gamma^h-1}\left(\theta^h h^{\gamma^h} - \left(1-\theta^h\right)(1-n)^{\gamma^h-1} n\right)\right]
\end{aligned}$$

After using the above expression for F and the expression for S from Lemma 1 and rearranging terms we get

$$G = 1 + \frac{\lambda(1-\theta^h)(1-n)^{\gamma^h}}{SD}\left(\frac{1}{1-n}\right)\left(\left(\gamma^h - \gamma^c\right)\theta^c c^{\gamma^c} + \gamma^h(1-\theta^c) D^{\gamma^c/\gamma^h}\right) \tag{A9}$$

This means that a sufficient condition for $G > 1$ is that $\gamma^h > \gamma^c$ and $\gamma^h \geq 0$. In addition, $0 < G < 1$ if $\gamma^h < \gamma^c$, $\gamma^h \leq 0$, and γ^h is not too small. The exact threshold for γ^h depends, among other things, on the value of λ (the multiplier of the implementability constraint). This proves Lemma 2. ■

Proof of Result 2. Assume now that $\gamma^c \neq \gamma^h$. Plugging (38) and (39) into (A7) gives

$$\begin{aligned}
\delta_h + r &= \left[\left(1-\tau^k\right) r + \delta_h + \tau^h r\right]\frac{G}{C} \Leftrightarrow \\
\tau^h - \tau^k &= \left(1 + \frac{\delta_h}{r}\right)\left(\frac{C}{G} - 1\right)
\end{aligned} \tag{A10}$$

By Lemma 2, $\gamma^h \neq \gamma^c$ implies that $\frac{C}{G} \neq 1$. Therefore, from (A10) it follows that $\tau^h \neq \tau^k$ if $\gamma^h \neq \gamma^c$.

We will prove part i) of Result 2 by showing that assumptions $\gamma^h > \gamma^c$ and $\gamma^h \geq 0$ imply that $\frac{C}{G} < 1$. From (A10) it then follows that $\tau^h - \tau^k < 0$. By Lemma 2, we know that assumptions $\gamma^h > \gamma^c$ and $\gamma^h \geq 0$ imply that $G > 0$. Therefore

$$\frac{C}{G} < 1 \Leftrightarrow G > C$$

By using the expressions for F and S from Lemma 1 and 2 and rearranging we get the following:

$$\begin{aligned}
G &> C \Leftrightarrow \\
1+\lambda\left(\gamma^h+\frac{\varphi F}{S}+\left(\gamma^c-\gamma^h\right)\frac{\theta^h h^{\gamma^h}-\left(1-\theta^h\right)(1-n)^{\gamma^h-1}n}{\theta^h h^{\gamma^h}+(1-\theta^h)(1-n)^{\gamma^h}}\right) &> 1+\lambda\left(\gamma^c+\frac{\varphi F}{S}\right) \Leftrightarrow \\
\gamma^h+\left(\gamma^c-\gamma^h\right)\frac{\theta^h h^{\gamma^h}-\left(1-\theta^h\right)(1-n)^{\gamma^h-1}n}{\theta^h h^{\gamma^h}+(1-\theta^h)(1-n)^{\gamma^h}} &> \gamma^c \Leftrightarrow \\
\gamma^h-\gamma^c-\left(\gamma^h-\gamma^c\right)\frac{\theta^h h^{\gamma^h}-\left(1-\theta^h\right)(1-n)^{\gamma^h-1}n}{\theta^h h^{\gamma^h}+(1-\theta^h)(1-n)^{\gamma^h}} &> 0 \Leftrightarrow \\
\left(\gamma^h-\gamma^c\right)\left[1-\frac{\theta^h h^{\gamma^h}-\left(1-\theta^h\right)(1-n)^{\gamma^h-1}n}{\theta^h h^{\gamma^h}+(1-\theta^h)(1-n)^{\gamma^h}}\right] &> 0 \Leftrightarrow \\
\left(\gamma^h-\gamma^c\right)(1-\theta^h)(1-n)^{\gamma^h}\left(1+\frac{n}{1-n}\right) &> 0 \qquad \text{(A11)}
\end{aligned}$$

Hence $G > C$. This proves part i) of Result 2.

We will prove part ii) of Result 2 by showing that assumptions $\gamma^h \leq 0$, $\gamma^h < \gamma^c$ and γ^h not too small imply that $\frac{C}{G} > 1$. By (A10), it then follows that $\tau^h - \tau^k > 0$. By Lemma 2, we know that in this case $0 < G < 1$ and therefore

$$\frac{C}{G} > 1 \Leftrightarrow C > G.$$

In addition, (A11) implies that

$$G < C \Leftrightarrow \left(\gamma^h-\gamma^c\right)(1-\theta^h)(1-n)^{\gamma^h}\left(1+\frac{n}{1-n}\right) < 0$$

Therefore, $C > G$. This proves part ii) of Result 2. ■