I. Introduction

This paper takes as given that (i) the burning of fossil fuel increases the carbon dioxide content in the atmosphere, which (ii) in turn leads to global heating and global climate change of a variety that, (iii) on net, is harmful to our welfare. To answer questions about the policy implications of this, a comprehensive and quantitative analysis of the two-way interaction between the economy, with its fossil fuel use, and the climate is necessary. In this paper, however, we focus on a particular aspect of this interaction: the role played by the industrial organization in the oil-producing sector of the world. Without a clear understanding of the world market for oil, the consequences of taxes and other policy instruments cannot be evaluated.

Analyses of the world oil market that are based on perfect competition and a finite amount of oil typically predict (i) that the oil price satisfies the Hotelling rule, i.e., increases so that the rate of return on storing oil is equal to the return on the capital market, (ii) that oil consumption follows a decreasing path, and (iii) that extraction is sequential in the sense that sources with lower extraction costs are depleted before high-cost sources are used. All these predictions are problematic to reconcile with data.

Our main point here is that the polar opposite case—where oil is supplied by a large agent with zero extraction costs who internalizes the effects of his decisions on all aggregates—seems useful for understanding historic and future developments in the oil market.

We also make another important assumption: drastic technology change will make oil superfluous at some future date when a “backstop technology” appears. We particularly focus on a case when the oil supplier optimally decides not to sell all her oil before date date. Thus, under our assumptions, it is as if the supply of oil were infinite. In contrast, under perfect competition, all oil with zero (or sufficiently low) extraction cost would necessarily be sold before the backstop technology appears. Our setting allows us to describe the determinants of the total amount of oil used, which is a key issue for climate policy.

Before the backstop technology is introduced, production is CES (has constant elasticity of substitution) across capital and energy. In contrast to Joseph E. Stiglitz (1976), we depart from unitary elasticity and argue, based on the analysis in John Hassler, Per Krusell, and Conny Olovsson (2009a), that an elasticity below one is the only fruitful way of interpreting the U.S. time series of input prices and quantities from the perspective that there is energy-saving technical change. The CES assumption is also very convenient for analytical tractability and for facilitating interpretation of the results.

To be as simple as our aim allows, we will consider a two-stage discrete-time model where oil is an essential input only in period zero (according to the mentioned CES function), and where a backstop technology using only capital as an input (in a linear fashion) is available from period two and on. Our analysis delivers laissez-faire world oil use along with its price and factor share, and we discuss how these depend on the primitives of the model; we also compare to the case with perfect competition and that where the finiteness of the oil endowment is binding. This analysis is contained in Section II, where we abstract from the economy-climate link. In Section III we then look at climate damages in

\[1\] It is straightforward to include labor in a natural way without changing the essence of the results.
a stylized way and derive optimal (energy-tax) policy given our model.

II. The benchmark model

We consider a two-country model where one agent, the representative leader of an oil cartel, or “the sheik” for short, has monopoly power over oil, but all other agents are price takers. The sheik thinks rationally about the consumption and savings decisions of his constituency, i.e., he internalizes the effects of his oil-producing decisions on the equilibrium choices of all small sheiks. Formally, we consider a Ramsey problem where the sheik is the planner. There is a finite amount of oil, $E$, available for extraction. At period 0, aggregate output, $Y$, is produced with capital $K$, and oil $E$, as inputs in a CES production function:

\[ Y = \left( (1 - \gamma)(AK)^{\epsilon-1} + \gamma(BE)^{\epsilon-1} \right)^{\frac{1}{\epsilon}}, \]

where $A$ denotes capital-augmenting technology and $B$ energy-augmenting, or energy-saving, technology. The parameter $\epsilon$ is the elasticity of substitution between the inputs (in efficiency units). Note that when $\epsilon = 0$, the production function is Leontief; when $\epsilon = 1$, it is Cobb-Douglas; and when $\epsilon = \infty$, $K$ and $E$ are perfect substitutes. The backstop technology, which arrives for use in period 1 and then remains available at all subsequent dates, is of the form

\[ Y = AK. \]

We assume that it is costless to extract the oil; in the present context, this is a simplification that is not entirely unrealistic. The utility function of all agents is assumed to be

\[ U = \sum_{t=0}^{\infty} \beta^t \log(c_t); \]

logarithmic curvature is not key for our results.

The market for final output is competitive, as are the input markets. Profit maximization then implies that factor prices equal marginal products:

\[ P = \gamma B \left( (1 - \gamma) \left( \frac{AK}{BE} \right)^{\frac{\epsilon-1}{\epsilon}} + \gamma \right)^{\frac{1}{\epsilon}}, \]

and

\[ R = (1 - \gamma) A \left( 1 - \gamma + \gamma \left( \frac{BE}{AK} \right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{1}{\epsilon}} \]

in period 0, and $R_t = A_t$, for all $t > 0$.

With no constraints on capital/credit markets, both the sheik and the rest of the world maximize present-value utility subject to a lifetime budget constraint. The lifetime budget constraint of the rest of the world is simply

\[ \sum_{t=0}^{\infty} c_{w,t} \prod_{s=0}^{t} \frac{1}{R_s} = R(K - k), \]

where $k$ is the capital owned by the sheik when the economy starts. Similarly, the budget constraint of the sheik reads

\[ \sum_{t=0}^{\infty} c_t \prod_{s=0}^{t} \frac{1}{R_s} = PE + Rk. \]

Agents can save in both capital and bonds. Total savings for the rest of the world in period $t$, denoted by $a_{w,t+1}$, are given by $q_t b_{w,t+1} + k_{w,t+1}$, where $q_t$ denotes the bond price and $b_w$ and $k_w$ are bond and capital holdings, respectively. Similarly, total savings for the sheik are given by $a_{t+1} = q_t b_{t+1} + k_{t+1}$. The bond is in zero net supply, i.e., $b'_w + b' = 0$.

The problem for the rest of the world is to maximize (3) by choice of $c_{w,t}$ and $a_{w,t+1}$ for all $t$ subject to (6). Similarly, the problem for any small sheik is to maximize (3) by choice of $c_t$ and $a_{t+1}$ for all $t$ subject to (7).

A. The sheik’s problem

Because of logarithmic utility, it follows straightforwardly that consumption of the sheik in period 0 is $c = (1 - \beta) (PE + Rk)$. Similarly, the consumption for the rest of the world in period 0 is $c_w = (1 - \beta) R(K - k)$. We first look at the case where the sheik initially owns no capital, i.e., the case $k = 0$. This means that the sheik maximizes oil revenues with respect to $E$, thus solving $\max PE$ subject to (4). Taking the first-order condition and solving for the ratio
\[
\frac{AK}{BE} = \left(\frac{1 - \gamma}{1 - \gamma - 1 - \epsilon} \right)^\frac{\epsilon}{\gamma}
\]

This equation determines \( E \) if there indeed is an interior solution. This occurs for all \( \epsilon < 1 \). For \( \epsilon \geq 1 \), there is no interior solution, the finiteness of the oil resource necessarily binds and the equilibrium amount of oil would be given by \( \bar{E} \).

Assuming \( \epsilon < 1 \) and that \( \bar{E} \) is sufficiently large to allow an interior solution, the supply of oil and aggregate output are then given by

\[
E = \frac{AK}{B} \left(\frac{\gamma}{1 - \gamma - 1 - \epsilon} \right)^\frac{\epsilon}{\gamma}
\]

and

\[
Y = AK \left(\frac{\epsilon}{1 - \gamma} \right)^\frac{\epsilon}{\gamma}
\]

Note that even though the production function is of the CES form, equilibrium output in an interior solution is effectively that of an “AK technology”. This is because in equilibrium, \( E \) is unconstrained and proportional to \( AK \).

Using (8) and (9) in (4) and (5), we have

\[
P = B (1 - \epsilon)^\frac{\epsilon}{\gamma} \gamma^\frac{\epsilon}{\gamma}
\]

and

\[
R = A \epsilon^\frac{\epsilon}{\gamma} (1 - \gamma)^\frac{\epsilon}{\gamma}
\]

Note, finally, given the equilibrium relations (9)-(11), that the monopoly share of output satisfies \( \frac{PE}{E} = 1 - \epsilon \). We summarize the most important of our findings as follows.

**PROPOSITION 1:** Under the assumptions stated above, if \( \epsilon < 1 \) and \( \bar{E} \) is large enough,

1) energy’s share of output is \( 1 - \epsilon \);
2) \( P \) is proportional to \( B \);
3) \( PE \) and \( Y \) are proportional to \( AK \) but independent of \( B \); and
4) \( E \) is proportional to \( AK \) and inversely proportional to \( B \).

Thus, if the elasticity of substitution across inputs is less than unitary, several striking facts emerge that are not present in a more standard case. In particular: (i) the oil use increases in the amount of capital available, (ii) the price of oil increases in the level of energy saving technology \( B \), and (iii) energy saving technology reduces oil use proportionally. In Hassler, Krusell, and Olovsson (2009b), we show that these features can be extended to an economy where oil is used in many periods. Then, oil use is influenced by how \( B, A \), and \( K \) vary over time; oil use increase over time when \( AK \) grows faster than \( B \). Furthermore, the price of oil does not satisfy Hotelling rule but is instead determined by the path of energy-saving technology.

**B. The sheik owns capital initially**

Now assume that the sheik owns some initial capital \( k \) when the economy starts. The problem for the sheik is equally simple, except in that he now needs to take into account the effect of oil supply on capital income in the initial period: the marginal product of capital, \( R \), increases in \( E \). Thus, if the sheik owns capital, there is an incentive to supply more oil than when he does not. With the sheik now maximizing \( PE + Rk \) with respect to \( E \), we can easily obtain first-order conditions (assuming \( \epsilon < 1 \) and an interior solution) and solve for the endogenous ratio \( \frac{AK}{BE} \), which satisfies

\[
\frac{AK}{BE} = \left(\frac{1 - \gamma}{1 - \gamma - 1 - \epsilon} \right)^\frac{\epsilon}{\gamma}
\]

Clearly, the ratio is strictly positive if \( \frac{1}{\gamma} < 1 - \epsilon \) and is zero when \( \frac{1}{\gamma} = 1 - \epsilon \). When the sheik has more than a fraction \( 1 - \epsilon \) of the total stock of capital, there is no interior solution to the sheiks problem and all existing \( \bar{E} \) would be supplied.

Under an interior solution, we now obtain

\[
E = \frac{AK}{B} \left(\frac{\gamma}{1 - \gamma - 1 - \epsilon} \right)^\frac{\epsilon}{\gamma}
\]

and equilibrium output as

\[
Y = AK \left(\frac{1 - \gamma}{K - k} \right)^\frac{\epsilon}{\gamma}
\]
Prices satisfy

\[ P = B \left( \frac{1 - \epsilon - \frac{k}{K}}{K} \right)^{1 \over 1-\gamma} \gamma^{1-\gamma} \]

and

\[ R = A \left( \epsilon \frac{K}{K - k} \right)^{1 \over 1-\gamma} (1 - \gamma)^{1 \over 1-\gamma}. \]

Finally, the monopoly share of output, \( \frac{PE}{1 - \epsilon - \frac{k}{K}} \), goes to zero as \( \frac{K}{k} \) approaches \( 1 - \epsilon \).

Thus, we see that ownership of capital will move the economy toward higher levels of oil supply and, in the version of the model without a climate externality considered in this section, toward the optimal oil use: \( \bar{E} \). In a dynamic version of the present economy where oil is used not just in the first period and where the oil sheik cannot commit to the levels of future oil production, the present model would actually give rise to (real) equilibrium indeterminacy. The reason is that if the small sheiks save in bonds, the return on savings is predetermined and there is no incentive to affect the marginal product of capital \( \text{ex post} \). If they save in physical capital, then \( \text{ex post} \) there is an incentive to increase the return on savings by increasing the oil supply. In equilibrium, the return on bonds and physical capital must be the same so small sheiks would be indifferent \( \text{ex ante} \) between assets, but different choices give different equilibrium oil use \( \text{ex post} \). Coordinated savings decisions, of course, would break the indeterminacy, as would the possibility of the oil-supplying sheik to commit \( \text{ex ante} \) to future levels of oil supply.

### C. Norway

The present model can be used to explain why countries, such as Norway, with high costs of oil extraction—in relative terms, at a point in time—produce oil at all. From a planner’s perspective, it would clearly be more efficient to extract all the cheap oil first. This point can be made in a general way in a dynamic model, but it is particularly simple to make the point in our essentially static setting: whenever there is zero-cost oil left in the ground, it does not make sense to extract oil elsewhere at a positive marginal cost. So what explains Norway? Our oil-monopoly model offers a simple answer. Suppose, first, that it is impossible for Norway to sell the oil-extraction rights to the sheiks. This, arguably, is a reasonable assumption given that the oil must be extracted on Norwegian territory and that it is hard to commit today to allowing this activity to be controlled by non-Norwegian nationals in the future. Suppose, moreover, that Norway is small, indeed infinitesimal, so that its oil extraction activity cannot influence world oil prices. Then our analysis above of the determination of prices and quantities remains unchanged. However, it also implies that Norway, which simply maximizes its income given world prices, will produce oil whenever the world price \( P \), which is strictly positive, exceeds its marginal cost of producing oil. Thus, a small fringe of oil producers will be producing alongside the sheiks, even though this activity is not optimal from the perspective of the world as a whole. In a dynamic setting, prices would, as noted above, grow at the rate of energy-saving technical change and Norway would sell its oil whenever the growth rate of the price minus extraction costs is lower than the interest rate.

### III. Climate damages

In this section, a negative externality from oil use is introduced. Following Nicholas Stern (2008) and Martin L. Weitzman (2007) we assume per-period utility of an additive form; for tractability, we assume unitary elasticity, giving

\[ \log c_t - \gamma_s \log S_t, \]

where \( S_t \) is the stock of \( \text{CO}_2 \) in excess of pre-industrial levels.\(^2\) The stock \( S_t \) follows

\[ S_{t+1} = (1 - \varphi) S_t + E_t, \]

which well approximates the medium- and long-run properties of the RICE carbon-cycle model (William D. Nordhaus and Joseph Boyer, 2000). Emissions only occur in period 0, implying \( S_t = \)

\(^2\)We assume that only the “rest of the world” is afflicted by the damage. To the extent the sheik internalizes the damage, which he would have to if it instead directly affected GDP, optimal carbon taxes would be lower and could turn into subsidies.
(1 - \varphi)^t E. Apart from an exogenous constant, the planner objective is therefore

\begin{equation}
(16) \quad \sum_{t=0}^{\infty} \beta^t (\log c_t - \gamma_s \log E_t).
\end{equation}

The per-period resource constraint is

\begin{equation}
(17) \quad c_t + K_{t+1} = Y_t,
\end{equation}

given \(Y_0\) from (1) and \(Y_t\) from (2) for all \(t > 0\).

### A. The social planning problem

The problem for the social planner is to maximize (16) subject to (17) by choice of \(E\) and sequences of \(c_t\) and \(K_{t+1}\). It is straightforward to verify that the first-order conditions for \(K_{t+1}\) imply \(K_{t+1} = \beta Y_t\) for all \(t\). Turning to energy choice, and ignoring the possibility of corner solutions, using the optimal savings rate we obtain

\begin{equation}
\gamma_B \frac{(1-\gamma)(\frac{AK}{BE})^{\frac{1}{\gamma_s}} + \gamma}{(1-\beta)Y} = \gamma_s E.
\end{equation}

This equation is straightforward to solve to obtain the optimal ratio

\begin{equation}
(18) \quad \frac{AK}{BE} = \left(\frac{1-\gamma}{\gamma} \frac{1}{(1-\beta)\gamma_s - 1}\right)^{\frac{1}{\gamma_s}}.
\end{equation}

This expression is positive so long as \(\gamma_s (1 - \beta) < 1\), in which case an increase in \(\gamma_s\) will decrease \(E\).

### B. Implementation of the first best

What instruments would a government use here? Taxation and quantity controls, on some abstract level, would be equivalent in the absence of constraints on the form that taxation/quantity controls would take. Here we will consider a unit tax \(\tau\) on oil use. Note, however, that an ad-valorem or oil-profit tax \(\hat{\tau}\) would be ineffective, since it would amount to profits given by \((1 - \hat{\tau})PE_t\), and net-of-tax-maximizing \(E\) would thus be the same as in laissez faire.

After individual savings decisions have been made, the sheik chooses \(E\) to solve \(\max_{\tau} (P - \tau) E\). We can solve for the \(\tau\) that implements optimal policy by using the first-order conditions for this problem evaluated at the optimal \(\frac{AK}{BE}\) ratio (i.e., that given by (18)). We thus obtain

\begin{equation}
(19) \quad \tau = \gamma^{\frac{1}{\gamma_s}} \frac{\frac{1 - \gamma_s (1 - \beta)}{\gamma_s (1 - \beta) + 1}}{\frac{1 - \gamma_s (1 - \beta)}{\gamma_s (1 - \beta) + 1} + 1}.
\end{equation}

It is straightforward to show that there exists a value for \(\gamma_s\) at which the monopoly outcome is optimal, so that the optimal tax is zero. Above it, the tax rate is increasing in \(\gamma_s\), as expected. Moreover, the optimal tax is increasing in \(\epsilon\).

### IV. Concluding remarks

Our analysis suggests that it is worthwhile taking monopoly power seriously in any analysis of the global energy market. Quantitative explorations are urgently needed now. These at least demand a model where oil is supplied dynamically, where there is a much larger “follower group” than just Norway, and where there is competition with alternative fossil-fuel sources, such as carbon. We explore the first of these issues in Hassler, Krusell, and Olovsson (2009b).

### References