The Role of Labour Markets for Fiscal Policy Transmission

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Abstract

This paper studies government spending shocks under alternative financing regimes in the presence of non-competitive labour markets.

The open economy New Keynesian DSGE model is extended by rigid labour markets of the Mortensen-Pissarides type and a detailed description of fiscal policy. Distortionary taxes provide a direct link between fiscal policy and the labour market by affecting the behavior of firms and workers on the matching market.

The results show that the nature of offsetting fiscal measures is critical for the effects of fiscal stimulus. Specifically, shifting the debt-stabilizing burden towards the distortionary labour tax makes the effects significantly more negative. A higher proportional tax rate raises the bargained wage to compensate workers for the otherwise falling net income. This effect causes vacancies and employment to fall and unemployment to rise.

The analysis suggests that a closer look at the functioning of labour markets may help to identify fiscal policy transmission channels not captured by the standard New Keynesian model.
1 Introduction

This paper studies fiscal policy in the presence of non-competitive labour markets. In order to address the question we extend the standard open-economy New Keynesian (NK) business cycle model in two dimensions: a detailed formulation of fiscal policy and labour market matching frictions along the lines of Mortensen and Pissarides (MP). We consider a small monetary union member state following Galí and Monacelli (2005). 1

We show that the detailed modelling of labour markets allows to identify fiscal policy transmission channels not captured by the standard NK models. The link between fiscal policy and the labour market is provided by distortionary labour taxes which directly influence the behavior of firms and workers on the matching market. On the other hand, labour market outcomes affect the budgetary stance through tax revenue and unemployment benefit expenditure.

Fiscal policy is back at the centre of the policy debate. At the same time, there is uncertainty both in the empirical and theoretical literature on what the effects of fiscal policy really are. The positive effect of increased government spending on output is widely acknowledged. But the magnitude of the output multiplier as well as effects on especially private consumption and the real wage are still debated. Private consumption, as the largest component of aggregate demand, is a key determinant of the size of the government spending multiplier.

The first generation of modern macroeconomic models focusing on real business cycles predicted that increased government spending would just crowd out private consumption2. In the New Keynesian model, the negative effect on private consumption is typically smaller because, when prices are rigid, firms increase labour demand as they respond to increased aggregate demand (see e.g. Linnemann and Schabert (2003)). The NK model in its standard form still predicts a negative response of private consumption to government spending shocks. The responses of employment and the real wage to fiscal shocks have received much less attention than effects on output and private consumption. In the RBC model, the increase in hours worked brought about by the negative wealth effect implied a lower marginal productivity of labour and a fall in real wages. In contrast, in the NK framework, the increase in labour demand together with the increase in labour supply can drive up the real wage or at least make it fall by a smaller amount.

Recent literature suggests that, in addition to price rigidities, the economy should be modelled as “non-Ricardian” to account for important transmission channels of fiscal policy. A “non-Ricardian” feature that has found to be important for the effects of fiscal policy, including the possibility of crowding-in consumption, is the inclusion of rule-of-thumb consumers who consume out of their current income (see e.g. Galí, Lopez-Salido and Valles (2007)). Coenen and Straub (2005), as well as Forni, Monteforte and Sessa (2009) have estimated the effects of fiscal policy in the Euro area with non-Ricardian consumers, and taking into account distortionary taxation. Distortionary taxation also breaks the Ricardian

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1 This paper is related to a larger modelling project where the objective is to build a framework for the macroeconomic analysis of the Finnish economy which would take into account specific country characteristics such as the requirements of Euro area membership, the wage negotiation tradition and rigid nominal wages as well as a fairly high labour taxation.

2 The standard RBC view is that the increase in government spending is perceived by intertemporally optimizing agents to increase taxes in the future and therefore to reduce the present value of after-tax income. This negative wealth effect causes private consumption to fall. (see e.g. Baxter and King, 1993).
equivalence. With proportional tax rates, the path of taxation does matter, since the tax rate affects the *intratemporal* optimality condition governing the consumer’s leisure-consumption trade-off.

In this paper, special emphasis is put on how the public debt resulting from a spending increase is paid back. As Baxter and King (1993) initially pointed out for the RBC model, the decision on how spending is financed is a crucial determinant for the sign and magnitude of the response to a government spending shock. In view of this early finding and compared to the large literature on the effects of fiscal stimulus, relatively little attention has been devoted to this question so far. We analyze the use of different tax rules to stabilize the debt. Both lump-sum and distortionary taxes are available to finance spending. Our main contribution is to show that the assumption of the offsetting fiscal measure is critical for the effects of fiscal stimulus, due to their different effects on the labour market.

Bilbiie and Straub (2005) consider different financing schemes of government spending and emphasize the labour market’s role in the propagation of fiscal policy shocks but they use a Walrasian labour market. The advantage of the matching model is that it explicitly accounts for the extensive margin of labour utilisation (the number of persons employed) and involuntary unemployment. To our knowledge, although the role of labour markets for the transmission of fiscal shocks has been recognised, the labour market matching model has not been used to identify these transmission channels in more detail. The REMS model for the Spanish economy does combine a labour market matching model and a range of tax instruments (see Boscá et al. (2007)) but they do not include wage rigidity and government spending is financed by lump-sum taxes.

The results of our baseline model with lump-sum taxes used to finance spending are in line with the majority of other New Keynesian models. Output increases, but the response of private consumption is negative. Most importantly, in our model, shifting the debt-stabilizing burden towards the distortionary labour tax has detrimental effects on the labour market outcome and on general economic performance. The wage bargaining model included in the labour market matching model implies that as soon as the tax rule becomes operative the higher proportional tax rate is internalised in the negotiation process. The bargained nominal wage rises to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs to firms which decrease open vacancies and unemployment starts rising. The fall in employment drives the private consumption response being more negative than when public debt is adjusted through lump-sum taxes.

In a closed economy, the effects of fiscal shocks depend, in addition to the above mentioned features, on the endogenous response of monetary policy that determines the evolution of real interest rates. In the case of the small currency union member state, the sign and qualitative pattern of the economy’s response to a change in government spending are less ambiguous because the nominal interest rate is fixed. Increased government spending at home raises output and the home price level, causing the real interest rate to fall. (see Galí and Monacelli (2005)).

The remainder of the paper is organised as follows: Section 2 describes the model, Section 3 describes the parametrization and steady state of the model, Section 4 presents the simulation results and elaborates on the transmission mechanisms of fiscal shocks, and Section 5 concludes.
2 The model

2.1 General features

The model considers a small monetary union member state and builds in this respect on Galí and Monacelli (2005). As in Corsetti, Meier and Müller (2009), however, we allow for a fraction of liquidity constrained consumers and close the model by assuming a debt-elastic interest rate. The home country is modelled along standard New-Keynesian practice comprising households, firms and a public sector. For simplicity, capital is not included as a factor of production.

The framework is augmented by a Mortensen and Pissarides (MP) search and matching labour market model (Mortensen and Pissarides, 1994; Pissarides 2000). Integrating matching frictions and Nash wage bargaining into the NK framework has been shown to improve the performance of the macro model by generating less cyclical wages and more cyclical employment than the competitive market-clearing model, thus bringing it closer to the data (see e.g. Woodford 2003).

For this paper, the structure of the standard labour market matching model has been amended with some key features that have, in more recent literature, been found useful in capturing the data and explaining the so-called unemployment volatility puzzle. There is an emerging consensus that labour market frictions, wage rigidities and staggered price setting together are needed to explain movements in unemployment, and the effects of monetary policy shocks (see e.g. Blanchard and Galí (2008), Christoﬀ el et al. (2008)). These features are taken to be important also for analyzing fiscal policy.

The present model adds rigidity in the adjustment of wages in the form of staggered bargaining initially developed by Gertler and Trigari (2006/2009), and applied in Gertler, Sala and Trigari (2008) and Christoﬀ el, Kuester and Linzert (2008). One advantage of this approach is that wage rigidity gets the explicit interpretation of longer wage contracts. Lengthening the duration of wage contracts makes wages in each period less responsive to economic conditions, and shifts adjustment to the labour quantity side.

In our framework, there is only one worker per firm, and the wage and price setting decisions are separated from each other. Labour market frictions arise in the intermediate good sector. The wholesale firms buy intermediate goods and re-sell them to the final goods sector. Wholesale firms operate under monopolistic competition and set prices subject to Calvo rigidities. Final goods are produced from domestic and imported intermediate inputs under perfect competition.

The other extension of the model concerns the public sector. The government policy instruments include a lump-sum tax, a proportional wage tax rate paid by the employees, wage taxes paid by the employers in the form of social security contributions, unemployment ben-

3Shimer (2005) argued that the MP model in its standard form does not sufficiently reproduce the relatively smooth behavior of wages and relatively volatile behavior of labor market variables observed in the data. Shimer argued that the problem arises because, in the standard model, the wage is renegotiated in every period by Nash bargaining and is thereby let to adjust very easily to changes in the economic environment. The volatility of wages absorbs a large part of the fluctuation that is actually observed in employment variables. In the growing body of literature that has attempted to explain the problem, also known as the unemployment volatility puzzle, the focus has accordingly been on ways to amplify the response of vacancies and unemployment to shocks. The range of alternative models proposed to solve the unemployment volatility puzzle include both flexible and rigid wage variants and have been summarized in e.g. Hall (2005).
efits and other government transfers as well as a consumption tax. The tax instruments react to changes in the debt-to-output ratio according to simple fiscal feedback rules. Government spending is subject to shocks.

2.2 Preferences

As in similar kinds of models, we adopt the representative or large household interpretation. This implies perfect consumption insurance, a key assumption needed to embed the MP model in a GE framework. Family members perfectly insure each other against variations in labour income due to their labour market status. This allows for the modelling of risk averse households and tackles the problem whereby households are identical but not all of their members are employed. As a result, the employment and unemployment rates are identical at the household level and across the population at large (see e.g. Merz (1995)).

It is assumed that a fraction \(1 - L\) of the families does not have access to asset markets and thus consumes its current income. The rest of the families (a fraction \(1 - L\) of the total) own firms, and may trade one-period bonds both domestically and internationally. These asset holding families will be indexed with a subscript \(tA\).

2.2.1 Asset Holders

Each asset-holding family maximizes its lifetime utility

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_{A,t} - \bar{C}_{A,t-1})^{1-\phi}}{1-\phi} - \delta \frac{(h_t)^{1+\phi}}{1+\phi} \right] \right\}
\]

where \(C_{A,t}\) is final good consumption by the representative asset-holding consumer in period \(t\), \(\bar{ \in (0, 1)}\) indicates an external habit motive, \(C_{A,t-1}\) stands for aggregate consumption of optimizing agents in the previous period, \(h_t\) are hours worked, and \(\delta\) is a scaling parameter of disutility of work, subject to the family budget constraint

\[
(1 + \tau^c_t) P_t C_{A,t} + B_{A,t} + B^*_{A,t} = n_t w_t h_t (1 - \tau_t) + (1 - n_t) P_t b
\]

\[
+ T R_t + R_{t-1}B_{A,t-1} + R^*_{t-1}p_b(b^*_{t-1}) B^*_{A,t-1} + D_{A,t} + n_t \Psi
\]

Consumption \(P_t C_{A,t}\) is subject to a proportional tax \(\tau^c_t\). The portfolio of financial assets includes domestic and foreign nominal one-period bonds \(B_{A,t}\) and \(B^*_{A,t}\). Domestic bonds are issued by the domestic government for which they represent debt. The interest rate paid or earned on foreign bonds by domestic households \(R^*_{t-1}p_b(b^*_{t-1})\) consists of the common currency union interest rate which for the small member state is taken to be exogenous and a country-specific risk premium. The two first terms on the right hand side describe income in the form of either wage or unemployment benefit. Income is also received as lump-sum

\[It is assumed that the capacity to participate in asset markets is the only difference between the two kinds of households. They supply an identical labor input and, therefore, the composition of households is the same across the economy. They all have a fraction \(n_t = 1 - u_t\) of employed members and a fraction \(u_t\) of unemployed workers.\]
transfers, in the form of repayment of last period’s bond purchases and nominal profit from firm ownership $D_t$. $n_t \Psi$ are fixed costs of production which accrue to the owners of the firms (see labour market section)

We leave aside for a moment the labour supply decision, which will be dealt with in the labour market subsection. The first order condition for domestic bond holdings is

$$\lambda_t = \beta E_t \left[ \lambda_{t+1} \frac{(1 + \tau^*_t)}{(1 + \tau^*_{t+1})} \frac{R_t}{\pi_{t+1}} \right]$$

which together with the marginal utility of consumption $\lambda_t = (C_{\lambda t} - \varkappa C_{\lambda t-1})^{-\eta}$ defines the intertemporal optimality condition for domestic consumption. $R_t = (1 + r^n_t)$ is the gross nominal return on domestic bonds, and $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is CPI inflation. The discount factor is the same for all optimizing agents in the economy\(^5\) and is defined throughout the paper as $\beta_{t,t+s} = \beta^s \frac{\lambda_{t+s}}{\lambda_t}$.

A similar Euler condition can be derived with respect to foreign bonds. As domestic and foreign bonds perfectly substitute each other, the arbitrage relation between them states that their nominal returns to the consumer have to be equal\(^6\), i.e.

$$R_t = R^*_t p(b^*_t)$$

where $p(b^*_t)$ is a risk premium on foreign bond holdings, which follows the function

$$p(b^*_t) = \exp \left[ -\gamma_{b^*} (b^*_t - \bar{b}) \right], \text{ with } \gamma_{b^*} > 0$$

The risk premium is assumed to be increasing in the aggregate level of foreign real debt as a share of domestic output ($b^*_t = \frac{B_t}{P_t Y_t}$)\(^7\). This should ensure the stability and determinacy of equilibrium in a small member state of the monetary union model\(^8\). In the steady state, the risk premium is assumed to be equal to one, and the domestic and foreign interest rates are the same. After loglinearization the arbitrage relation gets the form

$$\widehat{R}_t = \widehat{R}^*_t - \gamma_{b^*} \bar{b}^*_t$$

### 2.2.2 Non-asset holders

\(^5\)since the asset-holding families are assumed to own the firms.

\(^6\)This is a modified uncovered interest parity condition where no risk is associated with exchange rate movements, as both domestic and foreign bonds are denominated in euros.

\(^7\)This is one of the mechanisms suggested by Schmitt-Grohé and Uribe (2003) to close a small open economy model. Note that with the current notation a negative (positive) deviation of the stock of foreign bonds from the steady state zero level implies that the home country as a whole becomes a net borrower (lender), and faces a positive (negative) risk premium.

\(^8\)As Galí and Monacelli (2005) point out, along with accession to the monetary union the small member state no longer meets the Taylor principle since variations in its inflation that result from idiosyncratic shocks will have an infinitesimal effect on union-wide inflation, and will thus induce little or no response from the union’s central bank. According to the Taylor principle, in order to guarantee the uniqueness of the equilibrium, the central bank has to adjust the nominal interest rates more than one-for-one with changes in inflation (see e.g. Woodford (2003))
Non-asset holders do not optimize. Their consumption in nominal terms will just equal their current income in every period and they are therefore referred to as rule-of-thumb or hand-to-mouth consumers.\(^9\)

\[
(1 + \tau_t^c) P_t C_{L,t} = n_t w_t h_t (1 - \tau_t) + (1 - n_t) P_t b + TR_t
\]  

### 2.2.3 Aggregate consumption

Aggregate consumption is given by a weighted average of the corresponding variables for each consumer type

\[
C_t = LC_{L,t} + (1 - L) C_{A,t}
\]

### 2.3 The labour market

The labour market brings together workers and intermediate good firms. It was assumed that the capacity to participate in asset markets is the only difference between the two kinds of households. Therefore, both asset-holding and rule-of-thumb consumers supply an identical labour input, and the decisions of workers can be characterised from the point of view of a representative worker.

### 2.3.1 Unemployment, vacancies and matching

The measure of successful matches \(m_t\) is given by the matching function

\[
m_t(u_t, v_t) = \sigma_m u_t^\sigma v_t^{1-\sigma}
\]

where \(u_t\) and \(v_t\) are the aggregate measures of unemployed workers and vacancies. \(m_t\) is the flow of matches during a period \(t\), and \(u_t\) and \(v_t\) are the stocks at the beginning of the period. The matching function is, as usual, increasing in both vacancies and unemployment, concave, and homogeneous of degree one (see e.g. Petrongolo and Pissarides 2001). The Cobb-Douglas form implies that \(\sigma\) is the elasticity of matching with respect to the stock of unemployed people, and \(\sigma_m\) represents the efficiency of the matching process. The probabilities that a vacancy will be filled and that the unemployed person finds a job are respectively

\[
q_F^t = \theta t^F (\theta_t) = \frac{m_t}{v_t} = \sigma_m \theta_t^{-\sigma}
\]

\[
q_W^t = \theta t^W (\theta_t) = \frac{m_t}{u_t} = \sigma_m \theta_t^{1-\sigma}
\]

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\(^9\)Non-Ricardian households have been modelled in various ways in the literature. Some authors have assumed that they can hold money balances (e.g. Coenen et al. (2007)) and / or optimize intratemporally. Here they simply obey a rule to consume all of their current income. We are currently exploring the modelling of the intratemporal optimization problem of non-asset holders. This would have implications for the labour market matching model too, as the marginal utility term in the workers’ surplus equation would differ across different types of consumers.
and the inverse of these probabilities is the mean duration of vacancies and unemployment respectively.

\[ \theta_t = \frac{u_t}{u_t} \] is labour market tightness. The tighter the labour market is, or the less there are unemployed people relative to the number of open vacancies (i.e. larger \( \theta_t \)), the smaller the probability that the firm succeeds in filling the vacancy and the larger the probability that the unemployed person finds a job. In contrast, a decrease in the number of vacancies relative to unemployment (smaller \( \theta_t \)) implies that the unemployed person has a smaller probability to find a job.

In the beginning of each period, a fraction of matches will be terminated with an exogenous probability \( \rho_t \in (0, 1) \). The separation rate evolves as\(^{10}\)

\[
\log(\rho_t) = (1 - \nu_\rho) \log(\rho) + \nu_\rho \log(\rho_{t-1}) + \epsilon_t^\rho, \text{ where } \nu_\rho \in (0, 1), \epsilon_t^\rho \sim N(0, \sigma_\rho^2)
\]

Labour market participation can be characterised as follows. The size of the labour force is normalised to one. The number of employed workers at the beginning of each period is

\[
n_t = (1 - \rho_t) n_{t-1} + m_{t-1}
\]

where the first term on the right hand side represents those workers who were employed already in the previous period and whose jobs have survived beginning-of-period job destruction, and the second term covers those workers who got matched in the previous period and become productive in the current period. After the exogenous separation shock, the separated workers return to the pool of unemployed workers and start immediately searching for a job.

The number of unemployed is \( u_t = 1 - n_t \) respectively.

Job creation takes place when a worker and a firm meet and agree to form a match at a negotiated wage. The wage that the firm and the worker choose must be high enough that the worker wants to work in the job, and low enough that the employer wants to hire the worker. These requirements define a range of wages that are acceptable to both the firm and the worker. They do not, however, uniquely determine the equilibrium wage.

In the steady state an equal amount of jobs are created and destructed:

\[ JC = JD \iff m = \rho n \] \(^{12}\)

### 2.3.2 Wage bargaining

As in the standard Mortensen-Pissarides model, it is assumed that match surplus, the sum of the worker and firm surpluses is shared according to efficient Nash bargaining. In the efficient bargaining framework, wages and hours are negotiated simultaneously. The firm and the worker choose the wage and the hours of work to maximize the weighted product of their net return from the match.

\(^{19}\)Shocks to the job destruction probability have not been evaluated yet, so for now \( \rho \) can be treated as a constant probability of job destruction.
The structure of the staggered multiperiod contracting model follows Gertler, Sala and Trigari (2008). For comparison, the period-by-period bargaining outcome is presented in the appendix. The idea of staggered wage bargaining is analogous to Calvo price setting. Rigidity is created by assuming that a fraction $\gamma$ of firms are not allowed to renegotiate their wage in a given period. As a result, all workers in those firms receive the wage paid the previous period $w_{t-1}$ partially indexed to inflation. The constant probability that firms are allowed to renegotiate the wage is labeled $1 - \gamma$. Accordingly, $\frac{1}{1-\gamma}$ is the average duration of a wage contract. Thus, the combination of wage bargaining and Calvo price setting allows to give an intuitive interpretation to the source of wage rigidity instead of more of less ad hoc formulations in standard NK models. Period-by-period bargaining corresponds to the special case of $\gamma = 0$.

In each period, firms that are allowed to renegotiate their wage do it in the Nash bargaining set up. It is assumed that, as they become productive, new matches enter the same Calvo scheme for wage-setting than existing matches. Gertler and Trigari (2009) argue that after controlling for compositional effects there are no differences in the flexibility of new and existing worker’s wages$^{11}$.

The contract wage $w^*_t$ is chosen to solve:

$$\max [H_t (r)]^\eta [J_t (r)]^{1-\eta}$$

s.t.

$$w_{t+1} = \begin{cases} w_t \left[ \pi^w_t \left( \pi^{1-\varepsilon_w} \right) \right], & \text{with probability } \gamma \\ w^*_t, & \text{with probability } 1 - \gamma \end{cases}$$

where $H_t(r)$ and $J_t(r)$ are the worker and firm surpluses respectively, and $0 \leq \eta \leq 1$ is the relative measure of workers’ bargaining strength. The assumption of partial indexation of the past nominal wage to CPI inflation $\left( \pi_t = \frac{P_t}{P_{t-1}} \right)$ by $\left[ \pi^w_t \left( \pi^{1-\varepsilon_w} \right) \right]$ is similar to Smets and Wouters (2003) and Christoffel, Kuester and Linzert (2009).

The worker’s and the firm’s surplus from employment are the key determinants of the outcome of the wage bargain.

**Workers** The value to the worker of being employed consists of after-tax labour income, the disutility from working, and the expected present value of his situation in the next period.

$$W_t (r) = \frac{w^*_t h_t (1 - \tau_t)}{P_t} - g(h_t) \frac{\lambda_t}{\lambda_t} + E_t \beta_{t,t+1} (1 - \rho_{t+1}) \left[ \gamma W_{t+1} \left( w^*_t \left[ \pi^w_t \left( \pi^{1-\varepsilon_w} \right) \right] \right) + (1 - \gamma) W_{t+1}(w^*_{t+1}) \right] + E_t \beta_{t,t+1} \rho_{t+1} U_{t+1}$$

The value to the worker of being unemployed is

$^{11}$E.g. Pissarides (2007) and Haefke et al (2008) argue the opposite: that wages of newly hired workers are volatile unlike wages for ongoing job relationships. This would mean that there is wage rigidity, but not of the kind that leads to more volatility in unemployment fluctuations.
\[ U_t(r) = b + E_t \beta_{t,t+1} q_t^W \left[ \gamma W_{t+1} \left( w_t^* \left[ \pi_t^{\varepsilon w} \left( \pi^{1-\varepsilon w} \right) \right] \right) + (1 - \gamma) W_{t+1}(w_{t+1}^*) \right] \\
+ E_t \beta_{t,t+1} \left( 1 - q_t^W \right) U_{t+1} \]

where the first term on the RHS is the value of the outside option to the worker, i.e. the unemployment benefit \( b \), and the second term gives the expected present value of either working or being unemployed. Note that unemployed workers do not need to take into account the probability of job destruction even if they get matched because of the timing assumption. A match that has not yet become productive cannot be destroyed. Combining these value equations gives the expression for worker surplus

\[ H_t(r) = W_t(r) - U_t(r) \]

\[ = \frac{w_t^*}{P_t} h_t \left( 1 - \tau_t \right) - \frac{g(h_t)}{\lambda_t} - b \]

\[ + E_t \beta_{t,t+1} \left( 1 - \rho_{t+1} \right) \left[ \gamma H_{t+1} \left( w_t^* \left[ \pi_t^{\varepsilon w} \left( \pi^{1-\varepsilon w} \right) \right] \right) + (1 - \gamma) H_{t+1}(w_{t+1}^*) \right] \\
- q_t^W E_t \beta_{t,t+1} H_{x,t+1} \]

where the last term is the value for the worker to move from unemployment to employment.

Matching is a random process, i.e. there is no directed search in the labour market. New matches are subject to the same bargaining scheme as existing matches, and therefore the new worker does not have a priori knowledge of what the wage will be. He therefore values it according to its expected average value \( E_t H_{x,t+1} \).

**Intermediate firms** For the firm, the value of an occupied job is equal to the profit of the firm in the current period net of payroll taxes \( s_t \), and the expected future value of the job.

\[ J_t(r) = x_t f(h_t) - \frac{w_t^*}{P_t} h_t \left( 1 + s_t \right) - \Psi \]

\[ + E_t \beta_{t,t+1} \left( 1 - \rho_{t+1} \right) \left[ \gamma J_{t+1} \left( w_t^* \left[ \pi_t^{\varepsilon w} \left( \pi^{1-\varepsilon w} \right) \right] \right) + (1 - \gamma) J_{t+1}(w_{t+1}^*) \right] \]

where \( x_t \) is the relative price of the intermediate sector’s good, and \( f(h_t) = z_t h_t^\alpha \) is match output. The marginal product of labour is accordingly \( mpl_t = \alpha z_t h_t^{\alpha-1} = \alpha \frac{f(h_t)}{h_t} \). In addition to labour costs, the firm faces a per-period fixed cost of production \( \Psi \) which is independent of hours worked and defined in real terms. At the economy’s level, fixed costs are proportional to the number of employed workers. Labour-augmenting productivity \( z_t \) is identical for all matches and follows

\[ \log(z_t) = (1 - \nu_z) \log(z) + \nu_z \log(z_{t-1}) + \varepsilon_t^z \text{, where } \nu_z \in (0,1), \varepsilon_t^z \overset{iid}{\sim} N(0, \sigma_z^2) \]

The value to the firm of an open vacancy is
\[ V_t = -\kappa_t + E_t \beta_{t,t+1} q_t^F \left[ \gamma J_{t+1}(w_t^* \left( \pi_t^w \left( \pi_t^{1-\varepsilon_w} \right) \right)) + (1 - \gamma) J_{t+1}(w_{t+1}^*) \right] + E_t \beta_{t,t+1} (1 - q_t^F) V_{t+1} \]  

(18)

The value of a vacancy consists of an exogenous hiring cost \( \kappa_t \), and of the expected value from future matches. In equilibrium, all profit opportunities from new jobs are exploited so that the equilibrium condition for the supply of vacant jobs is \( V_t = 0 \). With each firm having only one job, profit maximization is equivalent to this zero-profit condition for firm entry. Setting the equation for \( V_t \) as zero in every period gives:

\[ \kappa_t = q_t^F E_t \beta_{t,t+1} \left[ \gamma J_{t+1}(w_t^* \left( \pi_t^w \left( \pi_t^{1-\varepsilon_w} \right) \right)) + (1 - \gamma) J_{t+1}(w_{t+1}^*) \right] \]  

(19)

This vacancy posting condition equates the marginal cost of adding a worker (real cost times mean duration of vacancy) to the discounted marginal benefit from a new worker. After taking into account the free entry condition, the firm surplus reduces to \( J_t \).

Total real profits of the intermediate sector firms which are paid to their owners i.e. Ricardian consumers are

\[ D_t^I = \int_0^{n_t} \left[ x_t z t h_{it}^\alpha - \frac{w_t^*}{P_t} h_{it} \left( 1 + s_t \right) - \Psi \right] di - \kappa_t v_t \]  

(20)

**Multiperiod bargaining set up** Unlike with period-to-period bargaining, in the presence of staggered contracting, firms and workers have to take into account the impact of the contract wage on the expected future path of firm and worker surplus. Accordingly, the first order condition for wage-setting is given by:

\[ \eta \Delta_t J_t (r) = (1 - \eta) \Sigma_t H_t (r) \]  

(21)

where the partial derivatives of the surplus equations w.r.t. the wage \( \Delta_t = P_t \frac{\partial H_t(r)}{\partial w_t} \) and

\[ \Sigma_t = -P_t \frac{\partial H_t(r)}{\partial w_t} \]

denote the effect of a rise in the **real** wage on the worker surplus and (minus) the effect of a rise in the real wage on the firm’s surplus respectively (see Appendix for details).

\[ \Delta_t = h_t (1 - \tau_t) + E_t \beta_{t,t+1} \left( 1 - \rho_{t+1} \right) \gamma \left[ \pi_t^v \left( \pi_t^{1-\varepsilon_w} \right) \right] \pi_t^{-1} \Delta_{t+1} \]  

(22)

\[ \Sigma_t = h_t (1 + s_t) + E_t \beta_{t,t+1} \left( 1 - \rho_{t+1} \right) \gamma \left[ \pi_t^v \left( \pi_t^{1-\varepsilon_w} \right) \right] \pi_t^{-1} \Sigma_{t+1} \]  

(23)

These expressions can be interpreted as the discounting factors for the worker and the firm (respectively) for evaluating the value of the future stream of wage payments. As wage contracts extend over multiple periods, agents have to take into account also the future probabilities of not being allowed to renegotiate the wage or of not surviving exogenous destruction. In the one firm - one worker setup used in this paper the discounting
factors would be equal across agents unless distortionary taxes were breaking this symmetry. With staggered bargaining, labour taxes enter the discounting factor equations of the agents implying that workers and firms also take into account the future path of taxation in their negotiating behaviour\textsuperscript{12}. In the limiting case of efficient bargaining, $\gamma = 0$, the partial derivatives of the surpluses w.r.t. the wage reduce to

$$t = h_t \left(1 - \tau_t\right),$$

and the first order condition accordingly reduces to its period-by-period counterpart $\eta \left(1 - \tau_t\right) J_t = (1 - \eta) \left(1 + s_t\right) H_t$.

Given that the probability of wage adjustment is i.i.d., and all matches at renegotiating firms end up with the same wage $w_t^*$, the evolution of the average hourly wage in the economy can be expressed as a convex combination of the contract wage and the average wage across the matches that do not renegotiate, after taking into account the indexation scheme.

$$w_{t+1} = (1 - \gamma) w_{t+1}^* + \gamma \int_{0}^{n_t} \frac{w_{t-1}(i)}{n_t} \left[\pi_t^{ew} \left(\pi_t^{1-ew}\right)\right] di$$

(24)

Wage dynamics The staggered bargaining framework has implications on the behavior of workers and firms. To describe wage dynamics in the presence of staggered contracting, we will develop loglinear expressions for the relevant wage equations in the same way as in Gertler, Sala and Trigari (2008). The contract wage is solved by first linearizing the first order condition

$$\hat{J}_t(r) + \hat{\Delta}_t = \hat{H}_t(r) + \hat{\Sigma}_t$$

(25)

and then plugging into the FOC the value equations and discounting factors for the worker and the firm respectively in their loglinearized form

$$\hat{H}_t = \frac{\bar{w} \bar{h} (1 - \bar{\tau}) \left(\hat{w}_t^* - \hat{P}_t + \hat{h}_t\right)}{H} - \frac{\bar{w} \bar{h} \tau}{H} \hat{\tau}_t - \frac{1}{1 + \phi} \frac{\bar{m} \bar{r} \bar{h}}{H} \left[\bar{m} \bar{r} s_t + \hat{h}_t\right]$$

$$- \bar{\beta} q^W E_t \left(\hat{q}_{t+1}^W + \hat{H}_{x,t+1} + \hat{\beta}_{t,t+1}\right)$$

$$+ \bar{\beta} (1 - \bar{\rho}) E_t \left(\hat{H}_{t+1} (w_{t+1}^*) + \hat{\beta}_{t,t+1}\right) - \bar{\beta} \bar{p} E_t \hat{\rho}_{t+1}$$

$$+ \frac{\bar{\beta} (1 - \bar{\rho}) \gamma}{1 - \bar{\beta} (1 - \bar{\rho}) \gamma} \frac{\bar{w} \bar{h} (1 - \bar{\tau})}{H} E_t \left(\hat{w}_t^* + \varepsilon_w \hat{\pi}_t - \hat{w}_t^{*+1} - \hat{\pi}_t^{+1}\right)$$

(26)

$$\hat{\Delta}_t = (1 - \iota) \hat{h}_t - \frac{(1 - \iota) \bar{\tau}}{1 - \bar{\tau}} \hat{\tau}_t + \iota E_t \left(\hat{\beta}_{t,t+1} + \varepsilon_w \hat{\pi}_t - \hat{\pi}_t^{+1} + \hat{\Delta}_t^{+1}\right) - \bar{\beta} \bar{p} \gamma E_t \hat{\rho}_{t+1}$$

(27)

\textsuperscript{12}In Gertler and Trigari (2009), differences in the worker’s and the firm’s optimization perspectives give rise to what they call the "horizon effect". These differences are caused by large firms taking into account possible changes in future hiring rates. The effect of distortionary taxes is different. Proportional tax rates influence the division of the total surplus from a job in equilibrium. More specifically, both the worker’s and the firm’s marginal tax rate effectively reduce the worker’s relative bargaining power, and consequently his share of the surplus irrespective of the bargaining horizon (see Pissarides (2000), chapter 9). Staggered bargaining just amplifies this effect.
and

\[
\hat{J}_t = \frac{\bar{m} p h}{\alpha J} \left( \hat{x}_t + \hat{m} p h_t + \hat{r}_t \right) - \frac{\bar{w} h (1 + \bar{s})}{J} \left( \hat{w}_t^* - \hat{P}_t + \hat{h}_t \right) - \frac{\bar{w} h \bar{s}}{J} \hat{s}_t \\
- \beta p E_t \hat{r}_{t+1} + \beta (1 - p) E_t \left( \hat{J}_{t+1} (w^*_{t+1}) + \hat{\beta}_{t,t+1} \right) \\
- \frac{\beta (1 - p) \gamma}{1 - \beta (1 - p) \gamma} \frac{\bar{w} h (1 + \bar{s})}{J} E_t \left( \hat{w}_t^* + \varepsilon_w \hat{p}_t - \hat{w}^*_{t+1} - \hat{p}_{t+1} \right)
\]

(28)

\[
\hat{\Sigma}_t = (1 - \iota) \hat{h}_t + \frac{(1 - \iota) \bar{w}}{(1 + \bar{s})} \hat{w}_t + \iota E_t \left( \beta_{t,t+1} + \varepsilon \hat{p}_t - \hat{p}_t - \hat{\Sigma}_{t+1} \right) - \beta p \gamma E_t \hat{r}_{t+1}
\]

(29)

The resulting contract wage is (see Appendix for details)

\[
\hat{w}_t^* = [1 - \iota] \hat{w}_t^0 (r) + \iota E_t (\hat{p}_{t+1} - \varepsilon \hat{p}_t) + \iota E_t \hat{w}_{t+1}^*
\]

(30)

where \( \iota = \beta (1 - p) \gamma \). This is the optimal wage set at time \( t \) by all matches that are allowed to renegotiate their wage. As is usual with Calvo contracting, it depends on a wage target \( w_t^0 (r) \) and next period’s optimal wage. As the probability of not being able to renegotiate the wage approaches zero \( \gamma \to 0, \iota \to 0 \), and the contract wage \( w_t^* \to w_t \), approaches the period-by-period Nash wage.

Gertler and Trigari (2006) show that compared to usual Calvo contracting, when the optimal wage is set as a result of a wage bargain, there are additional spillover effects on the target wage. The spillover effect of the wage bargain on the target wage arises because when wage contracts extend over multiple periods, the worker’s outside option depends also on the wage he or she can expect to earn elsewhere. The target wage equation can be written as

\[
\hat{w}_t^0 (r) = \hat{w}_t^0 + \varphi_H \Gamma E_t \left[ \hat{w}_{t+1} - \hat{w}_{t+1}^* \right]
\]

(31)

where \( \hat{w}_t^0 (r) \) is the target wage the firm and its worker would agree to if they are allowed to renegotiate and if firms and workers elsewhere remain on staggered multiperiod wage contracts, a sum of the wage that would arise if all matches were negotiating wages period-by-period \( \hat{w}_t^0 \) and the spillover effect\(^\text{13}\).

The spillover-free component of the target wage is of the same form than the period-by-period negotiated wage, adjusted for the multiperiod discounting factors.

\(^\text{13}\)In Gertler and Trigari’s (2006) original framework, there is an additional indirect spillover effect because the expected hiring rate of the large renegotiating firm affects the bargaining outcome. In the present one worker per firm setup that effect disappears.
\[\hat{w}_t^0 = \varphi_x (\hat{x}_t + \hat{mpl}_t) + \varphi_m \hat{mrs}_t + \varphi_H E_t \left( q_t^W + \hat{H}_{t+1} (w_{t+1}^\prime) + \hat{\beta}_{t,t+1} \right) \]
\[-\varphi_h \hat{h}_t - \varphi_s \hat{s}_t + \varphi_r \hat{r}_t + \varphi_D E_t \left[ \hat{\Sigma}_{t+1} - \hat{\Delta}_{t+1} \right] + \hat{P}_t \] (32)

The spillover coefficient \( \varphi_H \Gamma = \frac{(1-\eta) \beta q^w}{(1-\tau)} \) is positive indicating that when the expected average market wage \( E_t \hat{w}_{t+1} \) is higher than the expected contract wage \( E_t \hat{w}_{t+1}^* \) (i.e. unusually good labour market conditions) this raises the target wage in the negotiations. Unlike in the more standard monopolistic labour market modelling of New Keynesian DSGEs, here the spillover effect is responsible for a part of the added rigidity in wages. Thus, the wage inertia and resulting employment dynamics are not only a product of staggered multiperiod wage contracting, but also of the spillover effects from the Nash bargaining process.

Finally, combining all the relevant elements of the wage bargaining outcome yields the second-order difference equation for the evolution of the average wage (see Appendix)

\[\hat{w}_t = \lambda_b \left( \hat{w}_{t-1} + \varepsilon_w \hat{\pi}_{t-1} - \hat{\pi}_t \right) + \lambda_0 \hat{w}_t^0 + \lambda_f E_t \left( \hat{w}_{t+1} + \hat{\pi}_{t+1} - \varepsilon_w \hat{\pi}_t \right) \] (33)

Due to staggered contracting, \( \hat{w}_t \) depends on the lagged wage \( \hat{w}_{t-1} \), the spillover-free target wage \( \hat{w}_t^0 \), and the expected future wage \( E_t \hat{w}_{t+1} \).

### 2.3.3 Determining hours of work

While matches are restrained to renegotiate the wage only with a given exogenous probability, hours per worker can be renegotiated at each point in time. With efficient Nash bargaining, optimal hours of work can be found from the following first order condition obtained by differentiating the Nash maximand w.r.t hours

\[(1 - \tau_t) x_t f_{h,t} = (1 + s_t) \frac{g'(h_t)}{\lambda_t} \]

where \( f_{h,t} \) is, as before, the marginal product of the labour input i.e. hours, and which, using the expressions for the production and utility functions, can be written as

\[(1 - \tau_t) x_t mpl_t = (1 + s_t) mrs_t \] (34)

This optimality condition equates the value of marginal product to the marginal rate of substitution between work and leisure, and is thus of the same form as in a competitive labour market. At the margin, the hourly wage is such that the marginal cost to the worker from working is equal to the marginal gain to the firm but neither of these needs to be equal to the wage. Distortionary taxes drive an additional wedge between these two. The individual labour input of a worker is determined irrespective of the wage. But the model also allows for labour adjustment in the number of workers as defined by the vacancy posting condition and matching function. The matching model’s property that output can be changed by changing either the number of hours worked in each match \( h_t \) (the intensive margin) or the number of employed workers \( n_t \) (the extensive margin) is an important difference from the competitive model or most of the standard NK models.
2.4 Final good firms

There are two types of final goods firms. One is producing private consumption goods and the other type of final goods firm produces public consumption goods\(^{14}\).

2.4.1 Private consumption good

The private consumption good is a composite of intermediate goods distributed by a continuum of monopolistically competitive wholesale firms at home and abroad. Wholesale firms, their products and prices are indexed by \(i \in [0, 1]\). Final good firms operate under perfect competition and purchase both domestically produced intermediate goods \(y_{H,t}(i)\) and imported intermediate goods \(y_{F,t}(i)\). They minimize expenditure subject to the following aggregation technology

\[
C_t = \left(1 - W\right)^{\frac{1}{\omega}} \left( \frac{\int_0^1 y_{H,t}(i)^{\frac{\omega-1}{\omega}} di}{\int_0^1 y_{F,t}(i)^{\frac{\omega-1}{\omega}} di} \right)^{\frac{\omega}{\omega-1}} + W^{\frac{1}{\omega}} \left( \frac{\int_0^1 y_{F,t}(i)^{\frac{\omega-1}{\omega}} di}{\int_0^1 y_{F,t}(i)^{\frac{\omega-1}{\omega}} di} \right)^{\frac{\omega}{\omega-1}}
\]

where \(\omega\) measures the trade price elasticity, or elasticity of substitution between domestically produced intermediate goods and imported intermediate goods in the production of final goods for given relative prices, and \(W\) is the weight of imports in the production of final consumption goods. The parameter \(\varepsilon > 1\) is the elasticity of substitution across the differentiated intermediate goods produced and distributed within a country.

The optimization problem determining the allocation of expenditure between the individual varieties of domestic and foreign intermediate goods yields the following demand curves facing each wholesale firm

\[
y_{H,t}(i) = \left( \frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_{H,t}
\]

\[
y_{F,t}(i) = \left( \frac{p_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} Y_{F,t}
\]

where \(P_{H,t}\) and \(P_{F,t}\) are the aggregate price indexes for the domestic and foreign intermediate goods respectively

\[
P_{H,t} = \left[ \int_0^1 p_{H,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}
\]

\[
P_{F,t} = \left[ \int_0^1 p_{F,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}
\]

\(^{14}\)This is a standard assumption in New Open Economy Macro Models that assess fiscal policy. E.g. in Obstfeld and Rogoff’s (1996) extension of the Redux model, government spending is introduced as a basket of public consumption goods aggregated in the same way as for private consumption.
To determine the optimal allocation between the domestic and imported intermediate goods, the final good firm minimizes costs \( P_{H,t}Y_{H,t} + P_{F,t}Y_{F,t} \) subject to its production function or aggregation constraint. This yields the demands for the domestic and foreign intermediate good bundles by domestic final good producers

\[
Y_{H,t} = (1 - W) \left( \frac{P_{H,t}}{P_t} \right)^{-\omega} C_t
\]

\[
Y_{F,t} = W \left( \frac{P_{F,t}}{P_t} \right)^{-\omega} C_t
\]

where \( P_t \) is the home country’s aggregate price index, or consumption price index

\[
P_t = \left( (1 - W) P_{H,t}^{1-\omega} + W P_{F,t}^{1-\omega} \right)^{\frac{1}{1-\omega}}
\]

At the level of individual intermediate goods the law of one price holds\(^\text{15}\). That, together with the assumption that the weight of the home country good in the foreign consumer price index is infinitesimally small, implies that \( P_{F,t} \) is equal to the foreign CPI \( P_t^* \) (see Galí-Monacelli (2005)).

### 2.4.2 Public consumption good

The public consumption good is composed of only domestic intermediate goods \( g_t(i) \). This assumption implies, contrary to e.g. the Redux model, full home bias in government spending. This simplifying assumption can be supported by the observation from input-output tables that the use of foreign intermediate goods in government spending is significantly lower than in private consumption.

\[
G_t = \left[ \int_0^1 g_t(i) \frac{e^{1-\varepsilon}}{\varepsilon} \, di \right]^{\frac{\varepsilon}{1-\varepsilon}}
\]

Each wholesale firm \( i \) selling intermediate goods to the public consumption good producer faces the following demand schedule

\[
g_t(i) = \left( \frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} G_t
\]

\(^\text{15}\)Note, however, that due to home bias in consumption the basket of consumed goods may differ in the two areas, and therefore purchasing power parity does not hold.
2.5 Wholesale firms and price setting

The wholesale firms buy the homogeneous intermediate goods at nominal price \( p_{H,t} x_t \) per unit and transform them one-to-one into the differentiated product. The separation of the price setting decision from the wage setting decision is done to maintain the tractability of the model. Price rigidities arise at the wholesale level while search frictions and wage rigidity only directly affect the intermediate goods sector. This is a usual assumption in the models that extend the New Keynesian framework with Mortensen-Pissarides type labour markets (see e.g. Trigari (2006))\(^{16}\).

There is Calvo-type stickiness in price-setting and the relative price of intermediate goods \( x_t \) coincides with the real marginal cost faced by wholesale firms. In each period, the wholesale firm can adjust its price with a constant probability \( \frac{1}{1-\xi} \) which implies that prices are fixed on average for \( \frac{1}{1-\xi} \) periods. The wholesale firm’s optimization problem is to maximize expected future discounted profits by choosing the sales price \( p_{H,t}(i) \), taking into account the pricing frictions and the demand curve they face. It is assumed that the wholesale firm sells the home-country intermediate goods for the same price for domestic and foreign final goods producers, and for the domestic government.

The first order condition for the pricing decision of a wholesale firm that reoptimizes at \( t \) is

\[
E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ (1 - \varepsilon) \left( \frac{p_{H,t}(i)}{P_{H,t}} \right) y_{t+s}(i) - x_{t+s} y_{t+s}(i) \right] = 0
\]

(45)

where \( y_t(i) \) is the demand of firm \( i \)'s product by domestic private consumption good firms, foreign private consumption good firms and the domestic government as outlined in the previous section.

\[ y_t(i) = y_{H,t}(i) + y_{H,t}^*(i) + g_t(i) = \left( \frac{p_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y^D_t \]

where \( Y^D_t \) stands for total demand for domestic intermediate goods. All wholesale firms are identical except that they may have set their current price at different dates in the past. However, in period \( t \), if they are allowed to reoptimize their price, they all face the same decision problem and choose the same optimal price \( p_{H,t}^* \). Using the definition of the discount factor and rearranging, the FOC can be rewritten as

\[
E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[ \left( 1 - \varepsilon \right) \frac{p_{H,t}^*}{P_{H,t}} \left( \frac{1}{p_{H,t}^*} \right)^{-\varepsilon} Y^D_{t+s} \right] = 0
\]

(46)

which can be solved for \( \frac{p_{H,t}^*}{P_{H,t}} \) to yield the following pricing equation

\[
\frac{p_{H,t}^*}{P_{H,t}} = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} x_{t+s} \left( \frac{p_{H,t+s}}{P_{H,t+s}} \right)^{\varepsilon} Y^D_{t+s}}{E_t \sum_{s=0}^{\infty} \xi^s \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left( \frac{P_{H,t+s}}{P_{H,t+s}} \right)^{\varepsilon-1} Y^D_{t+s}}
\]

(47)

\(^{16}\)A number of extensions merge the intermediate and retail sectors so that there are interactions between wage and price setting at the level of the individual firm. E.g. Christoffel et al. (2009) assess the implications of that specification for inflation dynamics.
where $\frac{\varepsilon}{\varepsilon - 1} = \mu$ is the flexible-price markup. This is the standard Calvo result. In the absence of price rigidity, the optimal price would reduce to a constant markup over marginal costs. Log-linearizing the FOC around the steady state yields the New Keynesian Phillips Curve where domestic inflation depends on marginal costs and expected future inflation

$$\pi_{H,t} = \nu \hat{x}_t + \beta E_t \pi_{H,t+1}$$

where $\nu = \frac{(1-\xi)(1-\xi\beta)}{\xi}$.

Total real profits of the wholesale sector firms are

$$D_t^R = \int_0^{n_t} \left[ \left( \frac{P_{H,t}(i)}{P_{H,t}} - x_t \right) y_t(i) \right] di$$

2.6 Fiscal policies

The public sector’s role in this economy is to collect taxes and use them to finance unemployment benefits and lump-sum transfers as well as government spending $G_t$. The various tax instruments in use are the labour tax on workers $\tau_t$, payroll taxes on firms $s_t$, and a consumption tax $\tau^c_t$. Lump-sum transfers $TR_t$ may also be altered in response to changes in spending. The government also earns income through new debt issues in bonds which are repaid in the next period. The government budget constraint is

$$n_t w_t h_t(\tau_t + s_t) + \tau^c_t P_t C_t + B_t = P_{H,t} G_t + P_t bu_t + TR_t + R_{t-1} B_{t-1}$$

Accordingly, the government real debt $b_t = \frac{B_t}{P_t}$, evolves as

$$b_t = R_{t-1} \frac{b_{t-1}}{\pi_t} + \frac{P_{H,t}}{P_t} G_t + bu_t + \frac{TR_t}{P_t} - n_t \frac{w_t}{P_t} h_t(\tau_t + s_t) - \tau^c_t C_t$$

Fiscal policy is assumed to obey a rule whereby the chosen fiscal variable is adjusted to changes in debt as a fraction of steady state output. On the revenue side, we consider four alternative tax instruments: the lump-sum tax, consumption tax and the labour taxes on the employer and the employee. The rules relate the change in the policy instrument from its steady state level to the deviation of real debt from its target level

$$TAX_t = TAX + \omega_d \left( \frac{b_{t-1}}{Y_{t-1}} - \overline{b} \right)$$

where $TAX_t = \tau^LS_t, \tau^c_t, \tau_t, s_t$ and $\omega_d$ is the sensitivity of the tax instrument with respect to the government debt-to-output ratio. Government spending is characterised by the following autoregressive process

$$\log(G_t) = (1 - \rho_G) \log(\overline{G}) + \rho_G \log(G_{t-1}) + \epsilon^G_t, \text{ where } \rho_G \in (0, 1), \epsilon^G_t \sim N(0, \sigma^2_G)$$

where $\epsilon^G_t$ is the government spending shock.
2.7 Equilibrium

For each intermediate good, supply must equal total demand. The demand for good \( i \) is, as shown previously, \( y_t(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{\varepsilon} Y^D_t \), where \( Y^D_t \) is total demand for domestic intermediate goods by domestic and foreign final goods firms and the domestic government. Using the expressions for the demands for domestic intermediate good bundles derived previously, this can be written as

\[
y_t(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left\{ (1 - W) \left( \frac{P_{H,t}}{P_t} \right)^{-\omega} C_t + W \left( \frac{P_{H,t}}{P_t^*} \right)^{-\omega} C_t^* + G_t \right\}
\]

Following Galí and Monacelli (2005) defining an index for aggregate domestic demand

\[
Y^D_t = \left[ \int_0^1 y_t(i)^{\varepsilon-1} \, di \right]^{\frac{1}{\varepsilon}}
\]

allows us to rewrite this as

\[
Y^D_t = (1 - W) \left( \frac{P_{H,t}}{P_t} \right)^{-\omega} C_t + W \left( \frac{P_{H,t}}{P_t^*} \right)^{-\omega} C_t^* + G_t
\]

Aggregate domestic demand has to equal aggregate supply minus the resources lost to vacancy posting, leading to the home economy’s aggregate resource constraint

\[
Y_t = (1 - W) \left( \frac{P_{H,t}}{P_t} \right)^{-\omega} C_t + W \left( \frac{P_{H,t}}{P_t^*} \right)^{-\omega} C_t^* + G_t + \kappa_t v_t
\]

While the above equation states that in equilibrium domestic output has to equal its usage as consumption, exports and government spending, market-clearing in the intermediate good sector also requires

\[
Y_t = n_t z_t h_t^\alpha
\]

The net foreign asset position is determined by the trade balance - the difference between domestic output and domestic consumption.

\[
B_t^* - R^*_{t-1} \rho \left( b^*_t \right) B_{t-1}^* = P_{H,t} Y_t - P_t C_t - P_{H,t} G_t - P_{H,t} \kappa_t v_t
\]

This relation is obtained by combining the consumers’ budget constraint, the following aggregations, the government’s budget constraint and the economy’s aggregate resource constraint. Since only Ricardian consumers hold financial assets, the equilibrium equations for aggregate holdings of domestic and foreign bonds are

\[
(1 - L) B_{A,t} = B_t
\]

\[
(1 - L) B_{A,t}^* = B_t^*
\]

Similarly, only members of asset-holding families receive dividends from domestic firms
(1 – L) \( D_{A,t} = D_t \)

where total dividends accrued to households are the sum of the profits in the intermediate and wholesale sectors

\[
D_t = \left[ n_t x_t z_t h_t^\alpha - n_t \frac{w_t^*}{P_t} h_t (1 + s_t) - n_t \Psi \right] - \kappa_t v_t + (1 - x_t) y_t n_t
\]

\[
= n_t z_t h_t^\alpha - n_t \frac{w_t^*}{P_t} h_t (1 + s_t) - n_t \Psi - \kappa_t v_t
\]

\[
= Y_t - n_t \frac{w_t^*}{P_t} h_t (1 + s_t) - n_t \Psi - \kappa_t v_t
\]

(57)
3 Parameterization and steady state of the model

The parameter values are chosen mostly on the basis of existing literature, and are summarized in table 1. For preferences and the labour market part, they follow mainly Christoffel-Kuester-Linzert (2008) who use quarterly data from 1984Q1 to 2006Q4 for the euro area and for the open economy Corsetti, Meier and Müller (2009).

Table 1: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
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</thead>
<tbody>
<tr>
<td>Preferences</td>
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<tr>
<td>$\beta$</td>
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<td>Time-discount factor</td>
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<td>$\phi$</td>
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<td>Labour supply (Frish) elasticity $\frac{1}{\phi}$ of 0.2</td>
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<td>$\varphi$</td>
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<td>Risk aversion</td>
</tr>
<tr>
<td>$\kappa$</td>
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<td>External habit persistence</td>
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<td>$L$</td>
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<td>Labour market</td>
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<td>$\alpha$</td>
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<td>Labour elasticity of production</td>
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<tr>
<td>$\sigma$</td>
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<td>Elasticity of matches w.r.t. unemployment</td>
</tr>
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<td>0.5</td>
<td>Efficiency of matching</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.04</td>
<td>Exogenous quarterly job destruction rate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.6</td>
<td>Bargaining power of workers</td>
</tr>
<tr>
<td>$b$</td>
<td>0.43</td>
<td>Unemployment benefits</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.058</td>
<td>Vacancy posting costs</td>
</tr>
<tr>
<td>$z$</td>
<td>2.27</td>
<td>Technology, targets output $Y = 1$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.83</td>
<td>Pr(no renegotiation), avg duration of wage contracts of 6 qrts</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>0.24</td>
<td>Fixed cost of production</td>
</tr>
<tr>
<td>$\varepsilon_w$</td>
<td>0</td>
<td>Wage indexation; no indexation in baseline model</td>
</tr>
<tr>
<td>Wholesale sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>11</td>
<td>Elasticity of substitution, implies a markup of 10 percent</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.75</td>
<td>Calvo stickiness of prices, average duration of 4 qrts</td>
</tr>
<tr>
<td>$\nu \left( = \frac{(1-\varepsilon)(1-\beta \xi)}{\xi} \right)$</td>
<td>0.085</td>
<td>Coefficient of marginal costs in NK Phillips curve</td>
</tr>
<tr>
<td>Final goods sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(1 - \bar{W})$</td>
<td>0.75</td>
<td>Home bias in final goods production</td>
</tr>
<tr>
<td>$\bar{\omega}$</td>
<td>0.66</td>
<td>Trade price elasticity</td>
</tr>
<tr>
<td>$\gamma_{b^*}$</td>
<td>0.005</td>
<td>Debt-elasticity of interest rates</td>
</tr>
</tbody>
</table>

The quarterly discount factor is $\beta = 0.992$ which corresponds to an annual interest rate of 3.3%. The labour supply, or Frish elasticity ($\frac{1}{\phi}$), is set to 0.2. This is in the middle range of values implied by most microeconomic studies which estimate this elasticity to be between 0 and 0.5 (see Card (1994)) for a survey). Much higher elasticities have been generally used in the business cycle literature because macro elasticities account also for the variation in the employment rate\(^{17}\). The quarterly separation rate is calibrated at $\rho = 0.04$. The labour elasticity of production parameter is set to $\alpha = 0.66$ which implies decreasing returns to scale in the intermediate goods production sector, and a labour share of 60 percent. The

\(^{17}\)See Fiorito, R. - Zanella, G. (2008) for a recent comparison of micro and macro elasticities of labor supply. They estimate an individual elasticity of about 0.1 and an aggregate elasticity of about 1.
The unemployment benefit parameter is calibrated at \( b = 0.42 \) but generates a too high net replacement rate of 78 percent, defined as the ratio of net unemployment benefits to average net (after-tax) income from work \( \frac{b}{w_{th} (1 - \tau)} \). According to OECD’s "Benefits and Wages" publication, the average net replacement rate over 60 months of unemployment for Finland is 70 percent averaging over four different family types. The unemployment benefit is not assumed to be proportional to the wage nor to be indexed to inflation. As Christoffel et al. (2008) note, in labour market matching models, there is a trade-off between obtaining a reasonable labour share and a plausible replacement rate. Further, Costain and Reiter (2008) show that a real business cycle model augmented with labour market matching can be made consistent with either business cycle facts or the effects of labour market policies but not both. The assessment of the chosen parameters in the light of these considerations is left for future work.

The wholesale sector is calibrated in line with the literature so that the markup is at a conventional value of \( \mu = \frac{\varepsilon}{\varepsilon - 1} = 1.1 \). The Calvo parameter is \( \xi = 0.75 \) on the basis of CKL calibration from the Eurosystem Inflation Persistence Network. The average duration of prices is accordingly 4 quarters. As to wages, they are assumed to be renegotiated every one and a half years, implying \( \gamma = 0.83 \).

The steady state values of the model variables implied by the current parameterization can be found in table 2. The steady state equations of the model are in turn provided in appendix A. In the steady state, output is normalized to one, so that GDP components can be interpreted as shares of GDP. The working force is also normalised to one so that the steady state unemployment level is 9 percent. A symmetric open economy steady state is assumed where consumption levels are initially the same at home and abroad and both the trade balance and net foreign asset holdings are zero. As no capital is included in the model, output components private consumption and government consumption (and the tiny amount of resources lost to vacancy posting) are scaled so that private consumption accounts for 71 percent of steady state output and government consumption is 28 percent.

The steady state tax rates for labour and consumption are computed as ten year historical averages of corresponding tax rates in Finland times the model-implied tax base for that tax category. Accordingly, labour taxes for the employee and the employer respectively amount to 30 percent and 25 percent times the wage bill and the consumption tax rate corresponds to an average of 19 percent times the size of private consumption. The government’s steady state debt to GDP ratio is set at 45 percent.
### Table 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>$Y$</td>
<td>1</td>
<td>Output</td>
</tr>
<tr>
<td>$C$</td>
<td>0.72</td>
<td>Consumption</td>
</tr>
<tr>
<td>$u$</td>
<td>0.09</td>
<td>Unemployment rate</td>
</tr>
<tr>
<td>$kv$</td>
<td>0.003</td>
<td>Total vacancy costs</td>
</tr>
<tr>
<td>$n$</td>
<td>0.91</td>
<td>Employment</td>
</tr>
<tr>
<td>$qw$</td>
<td>0.4</td>
<td>Probability of finding a job</td>
</tr>
<tr>
<td>$qf$</td>
<td>0.7</td>
<td>Probability of finding a worker</td>
</tr>
<tr>
<td>$b/(wh(1 − τ))$</td>
<td>0.78</td>
<td>Net replacement rate</td>
</tr>
<tr>
<td>$nw$</td>
<td>0.60</td>
<td>Wage bill</td>
</tr>
</tbody>
</table>

**Fiscal policy**

| $τ_c$    | 0.13  | Consumption tax       |
| $τ$      | 0.18  | Labour tax rate on employee |
| $s$      | 0.15  | Employers’ social security contribution |
| $TR / τ^{LS}$ | 0.03 | Lump-sum tax |
| $d/Y$    | 0.45  | Government debt to GDP ratio |
| $G$      | 0.28  | Government spending   |
| $ρ_G$    | 0.8   | Autocorrelation of government spending |
| $ε_i^G$  | 0.05  | Government spending shock |

## 4 Model evaluation

In the following we analyze the transmission mechanism of fiscal policy under our labour market specification. Specifically, we assess the effects of government spending shocks because these are in the centre of the debate on the effects of fiscal policy. Special emphasis is put on how the public debt resulting from a spending increase is paid back. After being identified by Baxter and King (1993) as a crucial assumption for the effects of fiscal policy in an RBC model, the chosen financing scheme has more recently been recognised to shape the response to a government spending shock in a New Keynesian model as well (see Bilbiie and Straub (2004)). Other crucial assumptions, as pointed out by Galí et al. (2007) are the share of liquidity constrained consumers, the extent of price rigidity, the persistence of the government spending shock and the intertemporal path of taxation (i.e. how strongly and quickly taxes react to debt and deficit).

The point on offsetting fiscal measures has recently been taken up by Corsetti, Meier and Müller (2009) who analyze a policy that reduces spending over time in response to an initial rise in public debt. They find that this spending reversal enhances the expansionary impact of increased public spending. We analyze in the present paper the use of different tax rules to stabilize the debt although the framework would also allow to consider spending reversals. Tax instruments are assessed in isolation in order to identify the mechanisms at work with each instrument - instead of a more realistic scenario where fiscal policy consists of a combination of instruments - especially the interaction between labour market matching frictions and the chosen fiscal policy approach.

In the following simulations, the positive government spending shock generates public debt which is gradually paid back following alternative fiscal feedback rules. As a baseline,
we analyze an increase in government spending corresponding to an approximately 1 percent increase in aggregate demand with distortionary labour taxes and the consumption tax kept constant. The resulting public debt is brought back to its steady state level by allowing lump-sum taxes to increase as commonly assumed in most other papers. Then, to reveal the specific properties of the present model, two other tax instruments are considered: the labour tax on employees and the consumption tax. The results for flexible wages and with no rule-of-thumb consumers are displayed in Figure 1. The effects of wage rigidity and rule-of-thumb consumers for the results are assessed separately below for the baseline fiscal policy scenario.

4.1 The baseline response with a lump-sum tax rule

The baseline response to a positive government spending is qualitatively in line with that obtained from other New Keynesian models (see e.g. Linnemann and Schabert (2003), and for the specific effects on employment and the real wage Pappa (2009)). The rise in government demand has a positive effect on output. Because of full home bias in government consumption, the multiplier is directly proportional to the share of government spending in GDP and the size of the shock. The effect on private consumption is negative. The negative wealth effect causes an initial drop in private consumption and increases hours worked per person but the response of consumption is reversed by the increase in employment (the number of workers times individual hours).

The initial increase in aggregate demand raises the expected returns of firms from a filled vacancy. As is usual with labour market matching frictions, vacancies increase on impact but employment only starts to rise (unemployment starts to fall) from the next period on as new matches become productive. The combined increase in both labour demand and supply drives up the negotiated wage. Although not shown in the picture, also the real wage rises contemporaneously together with employment in line with recent findings by Pappa (2009).

The effect of increased government spending on the trade balance and on the terms-of-trade appear similar to what e.g. Kim and Roubini (2003) or Müller (2006) find. An increase in government spending appreciates the terms of trade and increases net exports. The terms of trade appreciation is natural in the presence of full home bias in government consumption: the export price index - which in this framework is the domestic price index (because of producer pricing) - rises relative to the foreign price index which is not affected by fiscal stimulus in the small member state. It should be noted that as the focus is on a small currency union member state there is no endogenous monetary policy response that would counteract the effect of rising prices in the home country.

As to the trade balance, there are two counteracting forces. On the one hand, the trade balance improves because the value of trade increases, but on the other hand higher prices of home-produced goods have a negative effect on the trade balance through the substitution channel. Here the former effect dominates. The latter effect tends to be larger the higher the home bias in private consumption and the higher the intratemporal elasticity of substitution between the home and foreign good.
Figure 1. The dynamic effects of a government spending shock: baseline vs. alternative debt-stabilizing fiscal rules. Note: baseline (rigid line), labour tax rule (dotted line), and consumption tax rule (dashed line).
4.2 Alternative fiscal policy scenarios

The results show that shifting the debt-stabilizing burden towards the distortionary labour tax (dotted line) significantly changes the picture. Most importantly, the wage bargaining framework included in the labour market matching model implies that as soon as the tax rule becomes operative the higher proportional tax rate is internalised in the negotiation process. The bargained nominal wage stays above its steady state level to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs to firms which decrease open vacancies and unemployment starts rising. The fall in employment drives the private consumption response being more negative than when public debt is adjusted through lump-sum taxes.

When consumption taxes are used to stabilize debt, the negative labour market reactions are absent although both tax increases imply a roughly equal initial revenue effect. Output and vacancies rise as much as with the lumpsum rule and government debt returns more quickly to its steady state level. Even the consumption response is less negative than in the case of the labour tax rule although consumption is directly taxed.

The results lend support to the argument on the importance of how the increased public spending is financed. The alternative scenarios assessed above indicate that a more detailed description of the labour market may bring forward important transmission channels of fiscal policy that have not received much attention in the existing literature.

We also investigated a similar government spending shock using the labour tax on the employer as the stabilizing instrument. The results are very similar than when using the fiscal rule on the employee’s labour tax. The only significant difference is that the negotiated wage does not rise in the same way leaving the worker’s net income and the firm’s labour cost approximately the same across scenarios. As labour costs are however raised directly by the tax on employers, the labour market outcome is similar with falling employment and rising unemployment. The simulations are available on request.

Automatic stabilizers are at work in the present setup. The initial expansion of output and the accompanying improvement in employment after a government spending shock increase the government’s labour tax revenues and decreases expenditure on unemployment benefits. However, consumption tax revenue falls as private consumption decreases and government debt increases significantly and persistently. Indeed, debt-stabilizing fiscal rules are needed to help bringing debt back to its steady state level in a reasonable time frame. Because of the small initial size of lump-sum taxes in the government budget compared to government spending, the increase in taxes dictated by that rule is relatively ineffective in restraining indebtedness unless a higher debt-sensitivity coefficient is assumed. The present rules are calibrated so that irrespective of the rule in force the initial fiscal policy tightening implied by the rule is approximately equal in all cases.

4.3 Staggered wage bargaining

Figure 2 shows the results for the baseline model where lump-sum taxes react to public debt with wages being negotiated, instead of period-by-period, on average once every sixth quarter.
Figure 2. The dynamic effects of a government spending shock: flexible wages (rigid line) vs rigid wages $\gamma = 0.83$ (dotted line).
Making the wage more rigid increases the magnitude of the responses of labour market variables. Vacancies react more strongly to the initial stimulus as firms’ expected profits are larger when labour costs do not rise. The more favourable labour market reaction in the short-term contributes to consumption falling less than in the baseline. However, in the longer term, as the wage adjusts upward, vacancies and employment start to fall and unemployment rises as shown by the right tails of the corresponding impulse response functions. Output remains lower than its steady state level and consumption below for a long time.

A recent contribution by Monacelli, Perotti and Trigari (2010) finds that (real) wage rigidity dampens the effect of government spending shocks on hiring, because although the total surplus from the match increases, the firm’s share of the surplus decreases discouraging hiring. One difference in their framework is that they do not combine wage rigidity with price rigidity.

4.4 The model with rule-of-thumb consumers

Following empirical evidence on the failure of the permanent income hypothesis (e.g. Deaton 1992), non-Ricardian behaviour in the form of a share of rule-of-thumb consumers has been introduced into New Keynesian DSGE models. Galí, Lopez-Salido and Vallés (2007) find that this specification can account for the evidence on positive consumption effects of government spending. The presence of consumers who consume all of their current income irrespective of possible future policies counteracts the typical negative wealth effect on private consumption. Galí et al. find, however, that the inclusion of non-Ricardian consumers accounts for the positive response of private consumption to increased government spending only when combined with imperfect labour markets.

The effects of including rule-of-thumb consumers in the present framework are shown in Figure 3. The assumption that one third of consumers cannot intertemporally optimize and just fully consume their current income, does create a strongly positive initial response of private consumption and increases significantly the output multiplier. This pattern of stronger initial response is seen also for other variables of interest. This effect is, however, short-lived because the offsetting lumpsum tax increase quickly begins to decrease the per-period income of rule of thumb consumers. It is assumed here that lumpsum taxes are levied on asset-holders and non-asset-holders at the same rate. Coenen and Straub (2005) found that the more the tax burden rests with Ricardian households the more government spending can crowd in private consumption. Also, the initial wage increase discourages the firms from hiring and vacancies turn negative which causes unemployment to rise. Government debt initially falls because all components of tax revenue rise.
Figure 3. The dynamic effects of a government spending shock. The rigid lines describe the baseline model. The dotted lines represent the model with rule-of-thumb consumers. Following existing literature, the share of these consumers is assumed to be one third.
5 Concluding remarks

This paper contributes to the ongoing debate about the effects of fiscal policy by analyzing government spending shocks under alternative fiscal rules and rigid labour markets. For this purpose, we have introduced fiscal policy and labour market matching frictions into an open-economy New Keynesian DSGE.

The link between fiscal policy and the labour market was introduced with the help of distortionary labour taxes which directly influence the behavior of firms and workers on the matching market. This approach provides a richer framework to analyse the effects of fiscal policy than in the existing literature e.g. by accounting for the extensive margin of labour adjustment and involuntary unemployment. Additional rigidity in wage determination was introduced with the help of Gertler and Trigari's staggered bargaining framework.

In our baseline model, the public debt resulting from increased government spending is paid back by allowing lump-sum taxes to increase. Results are in line with most existing New Keynesian literature. The government spending shock has an expansionary effect on output and a small but negative effect on private consumption. The latter is driven by the negative wealth effect but counteracted by a positive employment response brought about by increasing real wages and increasing labour supply along both the intensive and extensive margin.

The results show that the assumption of the offsetting fiscal measure is critical for the effects of fiscal stimulus. Specifically, shifting the debt-stabilizing burden towards the distortionary labour tax makes the effects significantly more negative. Most importantly, the wage bargaining model included in the labour market matching model implies that as soon as the tax rule becomes operative the higher proportional tax rate is internalized in the wage negotiation process. The bargained nominal wage rises to compensate workers for the otherwise falling net income. The higher wage directly implies higher labour costs to firms which decrease open vacancies and unemployment starts rising. The fall in employment implies a stronger contraction in private consumption compared with the more standard case of lump-sum tax adjustment.

Interestingly, the response of private consumption to fiscal stimulus is not as negative if consumption taxes are used to consolidate the debt. This is because they do not add distortions to the labour market and they are a large revenue component.

While the precise quantitative effects of our fiscal policy simulations are still work in progress, the main conclusion is rather general. In an economy with labour market rigidities, withdrawing fiscal stimulus by means of increased labour taxes has detrimental effects on growth and employment dynamics.

Some sensitivity analysis was made with respect to the inclusion of rule of thumb consumers. Our initial results indicate, in line with recent literature, that the presence of rule of thumb consumers significantly increases the effectiveness of fiscal policy. This feature of the model will be put more emphasis on in future work.

Similarly, while the main propositions of the Gertler and Trigari staggered bargaining framework were shown to hold also in the present setup, more effort will be put on exploring the role of wage rigidity for shock propagation. The assessment of the effects of a different wage bargaining regime will also be conducted as previous research indicates this could affect the results.
References


A Appendix

A.1 Steady state of the model economy

Euler equation

$$\beta = \frac{1}{R}$$

Marginal utility of consumption

$$\lambda = (C - \varkappa C)^{-\varepsilon}$$

Consumption of liquidity constrained consumers

$$C_L = \frac{1}{(1 + \tau)^\varepsilon} \left[ nwh (1 - \tau) + (1 - n) b + TR \right]$$

Aggregate consumption

$$C = (1 - L)C_A + LC_L$$

Interest rate on foreign bonds

$$R^* = R$$

FOC of retail firm

$$x = \frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon}$$

Matches

$$m = \sigma_m u^\sigma v^{1-\sigma}$$

Employment

$$\rho n = m$$

Unemployment

$$u = 1 - n$$

Probability of finding a worker

$$q^F = \frac{m}{v}$$
Probability of finding a job

\[ q^W = \frac{m}{u} \]

Labour market tightness

\[ \theta = \frac{v}{u} \]

FOC for hours

\[ (1 - \tau) xmpl = (1 + s) mrs \]

where

\[ mpl = \alpha z h^{a-1} \text{ and } mrs = \frac{\delta h^\phi}{\lambda} \]

Economy-wide resource constraint

\[ Y = (1 - W) C + WC^* + G + \kappa v = C + G + \kappa v, \text{ in the symmetric steady state} \]

Government budget constraint

\[ (1 - R) B = G + bu + TR - n wh (\tau + s) - \tau^C C \]

Market clearing / aggregate output

\[ Y = nz h^\alpha \]

Wage

\[ w = \frac{\eta}{(1 + s)} \left[ \frac{xmpl}{\alpha} + \frac{\kappa \theta}{h} - \frac{\Psi}{h} \right] + \frac{(1 - \eta)}{(1 - \tau)} \left[ \frac{mrs}{(1 + \phi) + \frac{b}{h}} \right] \]

Job creation condition

\[ \kappa = q^F \beta J \]

where the firm surplus

\[ J = \frac{1}{1 - \beta (1 - \rho)} [xzh^a - wh (1 + s) - \Psi] \]

Worker surplus

\[ H = \frac{1}{1 - \beta (1 - \rho - q^W)} \left[ wh (1 - \tau) - \frac{mrs h}{(1 + \phi) + b} \right] \]
Worker discount factor

$$\Delta = \frac{\bar{h}(1 - \bar{p})}{1 - \bar{\beta}(1 - \bar{p})\gamma}$$

Firm discount factor

$$\Sigma = \frac{\bar{h}(1 + s)}{1 - \bar{\beta}(1 - \bar{p})\gamma}$$
A.2 Model dynamics

The dynamics of the model are obtained by taking a log-linear approximation around a deterministic steady state.

Euler equation

\[ \hat{\lambda}_t = E_t \left( \hat{\lambda}_{t+1} + \hat{R}_t - \hat{\pi}_{t+1} \right) + \frac{\pi^c}{1 + \pi^c} \left( \hat{\pi}^c_t - \hat{\pi}^c_{t+1} \right) \]

Marginal utility

\[ \hat{\lambda}_t = -\frac{\theta}{(1 - \kappa)} \left( \hat{C}_{A,t} - \kappa \hat{C}_{A,t-1} \right) \]

Interest rates

\[ \hat{R}_t = \hat{R}^*_t - \gamma_b \hat{b}^*_t \]

Consumption of RT consumers

\[ \hat{C}_{L,t} = \frac{\pi \pi \tilde{h} (1 - \tau)}{(1 + \pi^c) \bar{C}_L} \left( \tilde{\hat{\pi}}_t + \hat{\pi}_t \right) + \frac{\pi \pi \tilde{h} (1 - \tau) - b}{(1 + \pi^c) \bar{C}_L} \hat{\pi}_t - \left( 1 - \frac{b (1 - \pi)}{(1 + \pi^c) \bar{C}_L} \right) \hat{P}_t \]

Aggregate consumption

\[ \hat{C}_t = \frac{L \bar{C}_L \hat{C}_{L,t}}{\bar{C}} + \frac{(1 - L) \bar{C}_A \hat{C}_{A,t}}{\bar{C}} \]

Matching function

\[ \hat{m}_t = \sigma \hat{u}_t + (1 - \sigma) \hat{v}_t \]

Employment dynamics

\[ \hat{n}_t = (1 - \rho) \hat{n}_{t-1} + \frac{\hat{m}}{\bar{n}} \hat{m}_{t-1} - \rho \hat{p}_t \]

Unemployment

\[ \hat{u}_t = -\frac{1 - \pi}{\bar{n}} \hat{n}_t \]
Transition probabilities
\[ q_t^F = \dot{m}_t - \dot{v}_t \]
\[ q_t^W = \dot{m}_t - \dot{u}_t \]

labour market tightness
\[ \ddot{\theta}_t = \dot{v}_t - \dot{u}_t \]

FOC for hours worked
\[
(1 - \overline{\tau})\, \overline{mpl} (\dot{x}_t + m\dot{p}_t) - \overline{xmpl} \ddot{r}_t = (1 + \overline{s}) \overline{mrs} m\dot{r}_t + \overline{smrs} \ddot{s}_t
\]
\[ \iff \] \[ \dot{x}_t = m\dot{r}_t - m\dot{p}_t + \frac{\overline{\tau}}{(1 - \overline{\tau})} \ddot{r}_t + \frac{\overline{s}}{(1 + \overline{s})} \ddot{s}_t \]

where
\[ m\dot{p}_t = z_t - (1 - \alpha) \dot{h}_t \]

and
\[ m\dot{r}_t = \phi \dot{h}_t - \dot{\lambda}_t \]

New Keynesian Phillips Curve
\[ \ddot{\pi}_{H,t} = \nu \dot{x}_t + \beta E_t \ddot{\pi}_{H,t+1} \]

where \( \ddot{\pi}_{H,t} = \ddot{P}_{H,t} - \ddot{P}_{H,t-1} \) is domestic inflation

First order condition for wage setting
\[ \ddot{J}_t(w^*_t) + \ddot{\Delta}_t = \ddot{H}_t(w^*_t) + \ddot{\Sigma}_t \]

Firm surplus
\[
\ddot{J}_t = \frac{\overline{mplh}}{\alpha J} (\dot{x}_t + m\dot{p}_t + \ddot{h}_t) - \frac{\overline{wh}(1 + \overline{s})}{\overline{J}} (\ddot{w}^*_t - \ddot{P}_t + \ddot{h}_t) - \frac{\overline{whs}}{\overline{J}} \ddot{s}_t
\]
\[ - \frac{\overline{\beta} \rho E_t \ddot{p}_{t+1} + \overline{\beta} (1 - \overline{\beta}) E_t \left( \ddot{J}_{t+1} (w^*_{t+1}) + \ddot{\beta}_{t,t+1} \right)}{1 - \overline{\beta}(1 - \overline{\beta}) \overline{\gamma}} \frac{\overline{w}(1 + \overline{s})}{\overline{J}} E_t \left( \ddot{w}^*_t + \varepsilon_w \ddot{w}_t - \ddot{w}^*_t - \ddot{w}^*_{t+1} \right) \]
Worker discount factor

\[
\hat{\Delta}_t = (1 - \iota) \hat{h}_t - \frac{(1 - \iota) \tau}{(1 - \tau)} \tilde{\tau}_t + \iota E_t \left( \tilde{\beta}_{t,t+1} + \varepsilon_w \tilde{\pi}_t - \tilde{\pi}_{t+1} + \hat{\Delta}_{t+1} \right) - \hat{\beta} \hat{\gamma} E_t \hat{\rho}_{t+1}
\]

Worker surplus

\[
\hat{H}_t = \frac{\bar{w} h (1 - \tau)}{H} \left( \tilde{w}_t^* - \tilde{P}_t + \hat{h}_t \right) - \frac{\bar{w} h \tau}{H} \tilde{\tau}_t - \frac{1}{1 + \phi} \frac{m_r \bar{w}}{H} \left( \tilde{m} \tilde{r}_t + \hat{h}_t \right) \\
- \hat{\beta} q^W E_t \left( \tilde{q}^W_t + \hat{H}_{x,t+1} + \tilde{\beta}_{t,t+1} \right) \\
+ \hat{\beta} (1 - \hat{\rho}) E_t \left( \hat{H}_{t+1} (w_{t+1}^*) + \tilde{\beta}_{t,t+1} \right) - \hat{\beta} \hat{\rho} E_t \hat{\rho}_{t+1} \\
+ \frac{\hat{\beta} (1 - \hat{\rho}) \gamma}{1 - \beta (1 - \hat{\rho}) \gamma} \frac{\bar{w} h (1 - \tau)}{H} E_t \left( \tilde{w}_t^* + \varepsilon_w \tilde{\pi}_t - \tilde{w}_{t+1}^* - \tilde{\pi}_{t+1} \right)
\]

Firm discount factor

\[
\hat{\Sigma}_t = (1 - \iota) \hat{h}_t + \frac{(1 - \iota) \bar{s}}{(1 + s)} \hat{S}_t + \iota E_t \left( \tilde{\beta}_{t,t+1} + \varepsilon_w \tilde{\pi}_t - \tilde{\pi}_{t+1} + \hat{\Sigma}_{t+1} \right) - \hat{\beta} \hat{\gamma} E_t \hat{\rho}_{t+1}
\]

Optimal contract wage

\[
\hat{w}_t^* = [1 - \iota] \bar{w}_t^0 (r) + \iota E_t (\tilde{\pi}_{t+1} - \varepsilon_w \tilde{\pi}_t) + \iota E_t \hat{w}_{t+1}^*
\]

Target wage

\[
\hat{w}_t^0 (r) = \hat{w}_t^0 + \varphi_H \Gamma E_t \left[ \hat{w}_{t+1} - \hat{w}_{t+1}^* \right]
\]

Spillover-free target wage

\[
\hat{w}_t^0 = \varphi_x \left( \tilde{x}_t + m \bar{p} \tilde{l}_t \right) + \varphi_m \bar{m} \bar{r} \tilde{s}_t + \varphi_H \Gamma E_t \left( \tilde{q}^W_t + \hat{H}_{t+1} (w_{t+1}^*) + \tilde{\beta}_{t,t+1} \right) \\
+ \varphi_h \hat{h}_t - \varphi_s \tilde{s}_t + \varphi_r \tilde{r}_t + \varphi_D E_t \left[ \hat{\Sigma}_{t+1} - \tilde{\Delta}_{t+1} \right] + \hat{P}_t
\]

Average wage

\[
\hat{w}_t = (1 - \gamma) \hat{w}_t^* + \gamma (\hat{w}_{t-1} - \tilde{\pi}_t + \varepsilon_w \tilde{\pi}_{t-1})
\]

or

\[
\hat{w}_t = \lambda_0 \hat{w}_{t-1} + \lambda_0 \hat{w}_t^0 + \lambda_f E_t \hat{w}_{t+1}
\]
Vacancy posting condition

\[
\hat{\kappa}_t - \hat{q}_t^F = E_t \left( \hat{J}_{t+1} (r) + \hat{\beta}_{t,t+1} \right) + \frac{\gamma}{1 - \tau} \hat{w}_{t+1} \left( \hat{w}_{t+1} + \hat{\pi}_{t+1} - \hat{\pi}_t - \varepsilon_w \hat{\pi}_t \right)
\]

Trade balance

\[
\hat{T}B_t = \hat{Y}_t - C \left( \hat{C}_t + \hat{P}_t - \hat{P}_{H,t} \right) - \hat{G}_t
\]

Economy-wide resource constraint (not updated)

\[
\hat{Y}_t = (1 - W) C \hat{C}_t + W C^* \hat{C}_t^* + (1 - W) C \hat{\pi}_t + W C^* \hat{\pi}_t^* - \left[ (1 - W) C \pi + W C^* \pi \right] \hat{P}_{H,t} + \hat{G}_t
\]

Consumer price index

\[
\hat{P}_t = (1 - W) \hat{P}_{H,t} + W \hat{P}_t^*
\]

Evolution of debt / Government budget constraint

\[
\hat{b}_t = \hat{R}b_t (\hat{R}_{t-1} + \hat{b}_{t-1} - \hat{\pi}_t) + C \left( \hat{P}_{H,t} - \hat{P}_t + \hat{G}_t \right) + b \bar{w}_t + \hat{T}R (\hat{T}R_t - \hat{P}_t)
\]

Market clearing / aggregate output (not updated)

\[
\hat{Y}_t + \hat{P}_t = \bar{h}_t \left( \hat{\pi}_t + \hat{\pi}_t^* + \alpha \hat{h}_t \right) - \hat{\nu} \left( \hat{\pi}_t + \hat{\pi}_t^* \right) + \hat{P}_{H,t}
\]
A.3 Period-by-period Nash bargaining

In the standard MP model, it is assumed that total match surplus, \( S_t = (W_t - U_t) + (J_t - V_t) \), the sum of the worker and firm surpluses is shared according to efficient Nash bargaining where wages and hours are negotiated simultaneously. The firm and the worker choose the wage and the hours of work to maximize the weighted product of the worker’s and the firm’s net return from the match.

\[
\max_{w,h} (H_t)^\eta (J_t)^{1-\eta}
\]

where \( 0 \leq \eta \leq 1 \) is the relative measure of workers’ bargaining strength.

The worker surplus gets the following form.

\[
H_t = W_t - U_t = \frac{w_t}{P_t} h_t (1 - \tau_t) - \frac{g(h_t)}{\lambda_t} - b + E_t \beta_{t,t+1} (1 - \rho_{t+1} - q_t^W) H_{t+1}
\]

and the firm surplus is (after taking into account the free entry condition \( V_t = 0 \))

\[
J_t = x_t f(h_t) - \frac{w_t}{P_t} h_t (1 + s_t) - \Psi + E_t \beta_{t,t+1} (1 - \rho_{t+1}) J_{t+1}
\]

The first-order condition for wage-setting is

\[
\eta \frac{\partial H_t}{\partial w_t} J_t = (1 - \eta) \frac{\partial J_t}{\partial w_t} H_t
\]

which would, without taxes, correspond to the simple surplus splitting result where the total surplus from the match is shared according to the bargaining power parameter \( \eta \).

The optimality condition for wage-setting can be rewritten as a wage equation that includes only contemporaneous variables by substituting the value equations into the Nash FOC, and making use of the expressions for the production and utility functions.

\[
\frac{w_t}{P_t} = \frac{\eta}{(1 + s_t)} \left[ \frac{x_t m p_l_t}{\alpha} - \frac{\Psi}{h_t} \right] + \frac{(1 - \eta)}{(1 - \tau_t)} \left[ \frac{m r s_t}{(1 + \phi)} + \frac{b}{h_t} + \frac{q_t^W}{h_t} E_t \beta_{t,t+1} H_{t+1} \right]
\]

where \( w_t \) is the nominal hourly wage in a match. Further using the Nash first order condition for next period and the job creation condition, allows to write it as

\[
\frac{w_t}{P_t} = \frac{\eta}{(1 + s_t)} \left[ \frac{x_t m p_l_t}{\alpha} + \frac{\kappa_t \theta_t}{h_t} - \frac{\Psi}{h_t} \right] + \frac{(1 - \eta)}{(1 - \tau_t)} \left[ \frac{m r s_t}{(1 + \phi)} + \frac{b}{h_t} \right]
\]

The wage equation is a convex combination of what the worker contributes to the match (the first square brackets) and what he has to give up in terms of disutility from supplying hours of work. Since workers and firms are homogeneous and all matches adjust their wages every period, they will all choose the same wage when they are allowed to negotiate. The economy’s wage bill is this wage rate times the total number of hours worked in the economy.
It is clear from the wage equation that the introduction of taxes works to decrease the worker’s relative effective bargaining power from $\frac{n}{(1+n)}$. Consequently, economic conditions get a smaller weight in wage determination.

A.3.1 Job creation under period-by-period bargaining

The dynamics of job creation under period-by-period bargaining are characterised as follows

$$\hat{\kappa}_t - \hat{q}_t^F = E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t + E_t\hat{J}_{t+1}$$

where $E_t\hat{J}_{t+1}$ can be found by first rewriting $E_tJ_{t+1}$ as (not updated)

$$E_tJ_{t+1} = x_{t+1}f(h_{t+1}) - w_{t+1}h_{t+1}(1 + s) + \frac{\kappa_{t+1}}{\hat{q}_{t+1}^F}$$

$$\iff E_t\hat{J}_{t+1} = \frac{\tilde{x}mplh}{\alpha J} (\tilde{x}_{t+1} + \tilde{mpl}_{t+1}) - \frac{\tilde{w}h (1 + \tilde{s})}{J} \tilde{w}_{t+1}$$

$$+ \left[ \frac{\tilde{x}mplh}{\alpha J} - \frac{\tilde{w}h (1 + \tilde{s})}{J} \right] \tilde{h}_{t+1} - \frac{\tilde{w}h s}{J} \tilde{s}_{t+1}$$

$$+ \frac{\beta (1 - \bar{p})}{\tilde{w}_{t+1}} (\hat{\kappa}_{t+1} - \hat{q}_{t+1}^F)$$

$$\Rightarrow \hat{\kappa}_t - \hat{q}_t^F = \left[ 1 - \frac{\beta (1 - \bar{p})}{\tilde{w}_{t+1}} \right] E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t + \frac{\tilde{x}mplh}{\alpha J} (\tilde{x}_{t+1} + \tilde{mpl}_{t+1})$$

$$- \frac{\tilde{w}h (1 + \tilde{s})}{J} \tilde{w}_{t+1} + \left[ \frac{\tilde{x}mplh}{\alpha J} - \frac{\tilde{w}h (1 + \tilde{s})}{J} \right] \tilde{h}_{t+1}$$

$$- \frac{\tilde{w}h s}{J} \tilde{s}_{t+1} + \frac{\beta (1 - \bar{p})}{\tilde{w}_{t+1}} (\hat{\kappa}_{t+1} - \hat{q}_{t+1}^F)$$

A.4 Dynamics with wage rigidity

The derivation of the wage under staggered contracting follows Gertler, Sala and Trigari (GST) (2008). The Nash first order condition is in this case

$$\eta \Delta_t J_t (w^*_t) = (1 - \eta) \sum H_t (w^*_t)$$

where the effect of a rise in the real wage on the worker’s surplus is
\[
\Delta_t = P_t \frac{\partial H_t (w_t)}{\partial w_t} \\
= h_t (1 - \tau_t) + E_t \beta_{t,t+1} (1 - \rho_{t+1}) \gamma P_{t+1} \frac{\partial H_{t+1} (w_t [\pi_t^w (\pi^{1-\varepsilon_w})])}{\partial w_t} \\
= h_t (1 - \tau_t) + E_t \beta_{t,t+1} (1 - \rho_{t+1}) \gamma (\pi_t^w (\pi^{1-\varepsilon_w})) h_{t+1} (1 - \tau_{t+1}) \\
+ E_t \beta_{t+1,t+2} \pi_{t+2} \gamma P_{t+2} \frac{\partial H_{t+2} (w_t [\pi_{t+1}^w (\pi^{1-\varepsilon_w})])}{\partial w_t} \\
= E_t \sum_{s=0}^{\infty} \beta_{t+1,t+3} \pi_{t+3} \gamma^s \left( \left( \frac{P_{t+1}}{P_{t-1}} \right) \right)^{\varepsilon_w (\pi^{1-\varepsilon_w})} h_{t+s} (1 - \tau_{t+s}) \\
\iff \Delta_t = h_t (1 - \tau_t) + E_t \beta_{t,t+1} (1 - \rho_{t+1}) \gamma [\pi_t^w (\pi^{1-\varepsilon_w})] \pi_{t+1}^{-1} \Delta_{t+1}
\]

And similarly for the firm

\[
\Sigma_t = -P_t \frac{\partial I_t (w_t)}{\partial w_t} = h_t (1 + s_t) + E_t \beta_{t,t+1} (1 - \rho_{t+1}) \gamma \left[ \pi_t^w (\pi^{1-\varepsilon_w}) \right] \pi_{t+1}^{-1} \Sigma_{t+1}
\]

The dynamic contract wage equation is solved by first linearizing the FOC for wage setting, and then substituting the linearized worker and firm surplus equations as well as the above discount factors in their loglinearized form (see GST (2008) for more details).

First order condition

\[
\hat{I}_t (w_t^*) + \hat{\Delta}_t = \hat{H}_t (w_t^*) + \hat{\Sigma}_t
\]

where the expressions for \( \hat{I}_t (w_t^*) \) and \( \hat{H}_t (w_t^*) \) can be found as follows

**Worker surplus** The worker surplus can be written as

\[
H_t (w_t^*) = \frac{w_t^*}{P_t} h_t (1 - \tau_t) - \left[ \frac{g (h_t)}{\lambda_t} + b + E_t \beta_{t,t+1} q_t^W H_{x,t+1} \right] \\
+ E_t \beta_{t,t+1} (1 - \rho_{t+1}) H_{t+1} (w_{t+1}^*) \\
+ \gamma E_t \beta_{t,t+1} (1 - \rho_{t+1}) \left[ H_{t+1} (w_t^* [\pi_t^w (\pi^{1-\varepsilon_w})]) - H_{t+1} (w_{t+1}^*) \right]
\]

In the last term, evaluate the expression \( E_t \left[ H_{t+1} (w_t^* [\pi_t^w (\pi^{1-\varepsilon_w})]) - H_{t+1} (w_{t+1}^*) \right] \)
When linearized, this expression gets the following form

\[
E_t \left[ \hat{H}_{t+1}(w^*_t \left[ \pi^{\text{ew}}_t (\pi^{1-\varepsilon_w}_t) \right]) - \hat{H}_{t+1}(w^*_{t+1}) \right] = \frac{\bar{w}h (1 - \tau)}{H} E_t \left[ \hat{w}^* + \varepsilon_w \hat{\pi}_t - \hat{w}^*_{t+1} - \hat{\pi}_{t+1} \right] + \beta (1 - \bar{p}) \gamma E_t \left[ \hat{H}_{t+2}(w^*_t \left[ \pi^{\text{ew}}_{t+1} (\pi^{1-\varepsilon_w}_{t+1}) \right]) - \hat{H}_{t+2}(w^*_{t+1} \left[ \pi^{\text{ew}}_{t+1} (\pi^{1-\varepsilon_w}_{t+1}) \right]) \right] + \beta (1 - \bar{p}) \gamma E_t \left[ \hat{H}_{t+2}(w^*_t \left[ \pi^{\text{ew}}_{t+1} (\pi^{1-\varepsilon_w}_{t+1}) \right]) - \hat{H}_{t+2}(w^*_{t+1} \left[ \pi^{\text{ew}}_{t+1} (\pi^{1-\varepsilon_w}_{t+1}) \right]) \right]
\]

Iterating forward this can be further simplified to yield

\[
E_t \left[ \hat{H}_{t+1}(w^*_t \left[ \pi^{\text{ew}}_t (\pi^{1-\varepsilon_w}_t) \right]) - \hat{H}_{t+1}(w^*_{t+1}) \right] = \frac{1}{1 - \beta (1 - \bar{p}) \gamma} E_t \left[ \hat{w}^* + \varepsilon_w \hat{\pi}_t - \hat{w}^*_{t+1} - \hat{\pi}_{t+1} \right]
\]

With the help of the above expression, the loglinear formulation of the worker surplus is found to be

\[
\hat{H}_t = \frac{\bar{w}h (1 - \tau)}{H} \left( \hat{w}^*_t - \hat{P}_t + \hat{h}_t \right) - \frac{\bar{w}h \bar{\pi}}{H} \hat{\tau}_t - \frac{1}{1 + \phi \frac{\bar{m}r_s h}{H}} \left[ \bar{m}r_s t + \hat{h}_t \right] - \beta q^W E_t \left( q^W_t + \hat{H}_{x,t+1} + \hat{\beta}_{t,t+1} \right) + \beta (1 - \bar{p}) \gamma E_t \left( \hat{H}_{t+1} \left( w^*_t + \hat{\beta}_{t,t+1} \right) - \bar{p} \hat{p} \hat{t}_{t+1} \right) + \frac{\beta (1 - \bar{p}) \gamma}{1 - \beta (1 - \bar{p}) \gamma} E_t \left( \hat{w}^*_t + \varepsilon_w \hat{\pi}_t - \hat{w}^*_{t+1} - \hat{\pi}_{t+1} \right)
\]

where as shown in Gertler and Trigari (2006) up to a first order approximation \( \hat{H}_{x,t+1} = \hat{H}_{t+1} (w_{t+1}) \).
Firm surplus  The firm surplus can be written as

\[ J_t(w_t^*) = x_t f(h_t) - \frac{w_t^*}{P_t} h_t (1 + s_t) + E_t \beta_{t,t+1} (1 - \rho_{t+1}) J_{t+1}(w_{t+1}^*) \]

\[ + \gamma E_t \beta_{t,t+1} (1 - \rho_{t+1}) \left[ J_{t+1}(w_t^* \ln \left( \frac{\pi_t^{\epsilon_w}}{\pi_t^{\epsilon_w}} \right)) - J_{t+1}(w_{t+1}^*) \right] \]

In the last term, evaluate the expression \( E_t \left[ J_{t+1}(w_t^* \ln \left( \frac{\pi_t^{\epsilon_w}}{\pi_t^{\epsilon_w}} \right)) - J_{t+1}(w_{t+1}^*) \right] \)

\[ = -E_t \left[ \frac{w_t^* \ln \left( \frac{\pi_t^{\epsilon_w}}{\pi_t^{\epsilon_w}} \right)}{P_{t+1}} - \frac{w_{t+1}^*}{P_{t+1}} h_{t+1} (1 + s_{t+1}) \right] \]

\[ + \gamma E_t \beta_{t,t+1,t+2} \left[ J_{t+2}(w_t^* \ln \left( \frac{\pi_{t+1}^{\epsilon_w}}{\pi_{t+1}^{\epsilon_w}} \right)) - J_{t+2}(w_{t+1}^* \ln \left( \frac{\pi_{t+1}^{\epsilon_w}}{\pi_{t+1}^{\epsilon_w}} \right)) \right] \]

When linearized this expression gets the following form

\[ E_t \left[ \tilde{J}_{t+1}(w_t^* \ln \left( \frac{\pi_t^{\epsilon_w}}{\pi_t^{\epsilon_w}} \right)) - \tilde{J}_{t+1}(w_{t+1}^*) \right] \]

\[ = -\frac{wh (1 + \bar{s})}{J} E_t \left[ \tilde{w}_t^* + \varepsilon_w \tilde{\pi}_t - \tilde{w}_{t+1}^* - \tilde{\pi}_{t+1} \right] \]

\[ + \bar{\gamma} (1 - \bar{\rho}) \gamma E_t \left[ \tilde{J}_{t+2}(w_t^* \ln \left( \frac{\pi_{t+1}^{\epsilon_w}}{\pi_{t+1}^{\epsilon_w}} \right)) - \tilde{J}_{t+2}(w_{t+1}^* \ln \left( \frac{\pi_{t+1}^{\epsilon_w}}{\pi_{t+1}^{\epsilon_w}} \right)) \right] \]

Iterating forward this can be further simplified to yield

\[ E_t \left[ \tilde{J}_{t+1}(w_t^* \ln \left( \frac{\pi_t^{\epsilon_w}}{\pi_t^{\epsilon_w}} \right)) - \tilde{J}_{t+1}(w_{t+1}^*) \right] \]

\[ = -\frac{1}{1 - \bar{\gamma} (1 - \bar{\rho}) \gamma} \left( \frac{wh (1 + \bar{s})}{J} E_t \left[ \tilde{w}_t^* + \varepsilon_w \tilde{\pi}_t - \tilde{w}_{t+1}^* - \tilde{\pi}_{t+1} \right] \right) \]

Finally, as with worker surplus, the loglinear formulation of the firm surplus can be found with the help of the above expression

\[ \tilde{J}_t = \frac{\bar{\gamma} (1 - \bar{\rho})}{\alpha J} \left( \tilde{x}_t + \tilde{m}_t + \tilde{h}_t \right) - \frac{wh (1 + \bar{s})}{J} \left( \tilde{w}_t^* - \tilde{P}_t + \tilde{h}_t \right) - \frac{wh \bar{s}}{J} \tilde{\pi}_t \]

\[ - \bar{\beta} \gamma E_t \tilde{\rho}_{t+1} + \bar{\beta} (1 - \bar{\rho}) E_t \left( \tilde{J}_{t+1}(w_{t+1}^*) + \tilde{\beta}_{t+1} \right) \]

\[ + \frac{\bar{\beta} (1 - \bar{\rho}) \gamma}{1 - \bar{\beta} (1 - \bar{\rho}) \gamma} \left( \frac{wh (1 + \bar{s})}{J} E_t \left( \tilde{w}_{t+1}^* + \tilde{\pi}_{t+1} - \tilde{w}_t^* - \varepsilon_w \tilde{\pi}_t \right) \right)\]
The Contract wage  Inserting the expressions for worker and firm surpluses into the FOC yields (after collecting the wage terms to the left-hand side and using the Nash FOC for next period)

\[
\begin{align*}
\Rightarrow \quad & \left[ \frac{wh}{H} (1 - \tau) + \frac{wh}{J} (1 + \bar{s}) \right] \hat{w}_t^* + \frac{\tau}{1 - \tau} \left[ \frac{wh}{H} (1 - \tau) + \frac{wh}{J} (1 + \bar{s}) \right] E_t \left( \hat{\bar{w}}_t + \varepsilon_w \hat{\bar{s}}_t - \hat{w}_{t+1}^* - \hat{\pi}_{t+1} \right) \\
& = \frac{\alpha \hat{\bar{w}}}{\alpha J} \left( \hat{x}_t + \frac{mpl_t}{J} \right) + \frac{1}{1 + \phi} \frac{\bar{m} \hat{\bar{w}} s_t}{\bar{H}} (\hat{m} \bar{s}_t) \\
& + \bar{\beta} (1 - \bar{p}) E_t \left[ \hat{J}_{t+1} (w_{t+1}^* + \bar{\beta}_{t,t+1}) - \bar{\beta} (1 - \bar{p}) E_t \left[ \hat{J}_{t+1} (w_{t+1}^* + \bar{\beta}_{t,t+1} + \hat{\Delta}_{t+1} - \hat{\Sigma}_{t+1} \right] + \bar{\beta} q^W E_t \left( \hat{q}_{t}^W + \hat{H}_{x,t+1} + \bar{\beta}_{t,t+1} \right) \\
& - \left[ \frac{wh}{H} (1 - \tau) + \frac{wh}{J} (1 + \bar{s}) \right] \hat{h}_t + \frac{wh s}{J} + \frac{(1 - \nu) \tau}{(1 + \bar{s})} \hat{s}_t + \frac{wh \tau}{H} - \frac{(1 - \nu) \tau}{1 - \tau} \hat{t}_t + \frac{wh (1 - \tau)}{H} + \frac{wh (1 + \bar{s})}{J} \hat{p}_t \\
& + \bar{\beta} (1 - \bar{p}) \gamma - \bar{\beta} (1 - \bar{p}) \right] E_t \hat{\Delta}_{t+1} - \left[ \bar{\beta} (1 - \bar{p}) \gamma - \bar{\beta} (1 - \bar{p}) \right] E_t \hat{\Sigma}_{t+1}
\end{align*}
\]

where \( \tau = \bar{\beta} (1 - \bar{p}) \). Noting that \( \left[ \frac{wh(1-\tau)}{H} + \frac{wh(1+\bar{s})}{J} \right] = \frac{wh(1+\bar{s})}{\eta} = \frac{wh(1-\tau)}{(1-\eta)H} \), and using the steady state equations for \( \hat{\Delta} \) and \( \hat{\Sigma} \), and the Nash FOC allows us to rewrite the contract wage equation in the following simpler form

\[
\begin{align*}
\Rightarrow \quad & \hat{w}_t^* + \frac{\tau}{1 - \tau} E_t \left( \hat{\bar{w}}_t + \varepsilon_w \hat{\bar{s}}_t - \hat{w}_{t+1}^* - \hat{\pi}_{t+1} \right) \\
& = \varphi_x \left( \hat{x}_t + \frac{mpl_t}{J} \right) + \varphi_m \hat{\bar{w}} s_t + \varphi_H E_t \left( \hat{q}_{t}^W + \hat{H}_{x,t+1} + \bar{\beta}_{t,t+1} \right) \\
& - \varphi_h \hat{h}_t - \varphi_s \hat{s}_t + \varphi_t \hat{t}_t + \varphi_D E_t \left[ \hat{\Sigma}_{t+1} - \hat{\Delta}_{t+1} \right] + \hat{p}_t \\
& = \hat{w}_t^0 (r)
\end{align*}
\]

where \( \hat{w}_t^0 (r) \) is the target wage in the bargain, and its coefficients are

\[
\begin{align*}
\varphi_x &= \frac{\alpha \hat{\bar{w}}}{\alpha w (1 + \bar{s})}, \quad \varphi_m = \frac{\bar{m} \hat{\bar{w}} s_t}{w (1 - \tau)}, \quad \varphi_H = \frac{(1 - \eta) \hat{H}}{wh (1 - \tau)} \bar{q}^W \\
\varphi_h &= \left\{ 1 - \frac{\alpha \hat{\bar{w}}}{\alpha w (1 + \bar{s})} - \frac{\bar{m} \hat{\bar{w}} s_t}{w (1 - \tau)} \right\}, \quad \varphi_s = \frac{\eta \hat{s}_t}{(1 + \bar{s})} \left[ 1 + \frac{(1 - \nu) \hat{J}}{wh (1 - \tau)} \right] \\
\varphi_t &= \frac{(1 - \eta) \tau}{(1 - \tau)} \left[ 1 - \frac{(1 - \nu) \hat{J}}{wh (1 - \tau)} \right], \quad \text{and} \quad \varphi_D = \frac{\bar{\beta} (1 - \bar{p}) (1 - \gamma) \eta \hat{J}}{wh (1 + \bar{s})}
\end{align*}
\]

The target wage \( \hat{w}_t^0 (r) \) is of the same form than the period-by-period negotiated wage, adjusted for the new bargaining weights. The equation for the contract wage can be further rewritten as
\[
\frac{1}{(1-\nu)} \hat{w}_t^* = \hat{w}_t^0(r) + \frac{\nu}{(1-\nu)} E_t (\hat{\pi}_{t+1} - \hat{\epsilon}_w \hat{\pi}_t) + \frac{\nu}{(1-\nu)} E_t \hat{w}_{t+1}^*
\]
\[
\iff \hat{w}_t^* = [1-\nu] \hat{w}_t^0(r) + \nu E_t (\hat{\pi}_{t+1} - \hat{\epsilon}_w \hat{\pi}_t) + \nu E_t \hat{w}_{t+1}^*
\]

This is the optimal contract wage set at time \(t\) by all matches that are allowed to renegotiate their wage. As is usual with Calvo-type contracting, it depends on a wage target \(w_t^0(r)\) and next period’s optimal wage.

**The spillover effect** To derive the spillover effect, consider the worker surplus with optimal (contract) wage versus the expected average market wage in the same way as above

\[
E_t \hat{H}_{t+1}(w_{t+1}) = E_t \hat{H}_{t+1}(w_t^*) + \frac{W \nu (1-\pi)}{(1-\nu)H} E_t (\hat{w}_{t+1} - \hat{w}_{t+1}^*)
\]

Denoting \(\frac{W \nu (1-\pi)}{(1-\nu)H} = \Gamma\) and substituting the above expression in the target wage equation gives

\[
\hat{w}_t^0(r) = \varphi (\hat{x}_t + \hat{m} \hat{p}_t) + \varphi_m \hat{m} \hat{s}_t + \varphi_H E_t \left( \hat{\nu}_t + \hat{H}_{t+1}(w_t^*) + \beta_{t,t+1} + \Gamma E_t [\hat{w}_{t+1} - \hat{w}_{t+1}^*] \right) + \varphi_{ht} \hat{h}_t + \varphi_{st} \hat{s}_t + \varphi_{t} \hat{\pi}_t + \varphi_{D} E_t \left[ \hat{\sigma}_{t+1} - \hat{\Delta}_{t+1} \right] + \hat{P}_t
\]
\[
\iff \hat{w}_t^0(r) = \hat{w}_t^0 + \varphi_H \Gamma E_t [\hat{w}_{t+1} - \hat{w}_{t+1}^*]
\]

where the target wage \(\hat{w}_t^0(r)\) - the wage the firm and its worker would agree to if they are allowed to renegotiate, and if firms and workers elsewhere remain on staggered multiperiod wage contracts - is a sum of the wage that would arise if all matches were negotiating wages period-by-period \(\hat{w}_t^0\) and the spillover effect \(\varphi_H \Gamma E_t [\hat{w}_{t+1} - \hat{w}_{t+1}^*]\).

**Evolution of the average wage** To derive the appropriate loglinear expression for the evolution of the average wage, first collect the necessary elements from previous calculations

1) The contract wage

\[
\hat{w}_t^* = [1-\nu] \hat{w}_t^0(r) + \nu E_t (\hat{\pi}_{t+1} - \hat{\epsilon}_w \hat{\pi}_t) + \nu E_t \hat{w}_{t+1}^*
\]

2) The average wage

\[
\hat{w}_t = (1-\gamma) \hat{w}_t^* + \gamma (\hat{w}_{t-1} - \hat{\pi}_t + \epsilon_w \hat{\pi}_{t-1})
\]
3) The target wage
\[
\hat{w}_t^0 (r) = \hat{w}_t^0 + \varphi_H \Gamma E_t \left[ \hat{w}_{t+1} - \hat{w}_{t+1}^* \right]
\]

First, insert the target wage in the contract wage equation
\[
\hat{w}_t^* = [1 - \lambda] \left( \hat{w}_t^0 + \varphi_H \Gamma E_t \left[ \hat{w}_{t+1} - \hat{w}_{t+1}^* \right] \right) + \tau E_t (\hat{\pi}_{t+1} - \varepsilon_w \hat{\pi}_t) + \tau E_t \hat{w}_{t+1}^*
\]

Then update the average wage equation by one period and take expectations
\[
E_t \hat{w}_{t+1} = (1 - \gamma) E_t \hat{w}_{t+1}^* + \gamma (\hat{w}_t - E_t \hat{\pi}_{t+1} + \varepsilon_w \hat{\pi}_t)
\]

\[\iff \] \[E_t \hat{w}_{t+1} = \frac{1}{(1 - \gamma)} \left( E_t \hat{w}_{t+1}^* - \gamma (\hat{w}_t - E_t \hat{\pi}_{t+1} + \varepsilon_w \hat{\pi}_t) \right) \]

Use this expression to eliminate \(E_t \hat{w}_{t+1}^*\) from the contract wage equation
\[
\hat{w}_t^* = [1 - \lambda] \left( \hat{w}_t^0 + \varphi_H \Gamma E_t \hat{w}_{t+1} - \varphi_H \Gamma \left[ \frac{1}{(1 - \gamma)} \left( E_t \hat{w}_{t+1} - \gamma \hat{w}_t + \gamma E_t (\hat{\pi}_{t+1} - \varepsilon_w \hat{\pi}_t) \right) \right] \right) + \tau E_t (\hat{\pi}_{t+1} - \varepsilon_w \hat{\pi}_t) + \tau E_t \hat{w}_{t+1}
\]

\[\iff \] \[\hat{w}_t^* = (1 - \lambda) \hat{w}_t^0 + (1 - \lambda) \varphi_H \Gamma E_t \hat{w}_{t+1} - \lambda \varphi_H \Gamma \frac{1}{1 - \gamma} E_t \hat{w}_{t+1}
\]

\[\iff \] \[\hat{w}_t^* = (1 - \lambda) \hat{w}_t^0 + \frac{\lambda}{1 - \gamma} \left( \hat{w}_t - E_t \hat{\pi}_{t+1} + \varepsilon_w \hat{\pi}_t \right) + \tau E_t (\hat{\pi}_{t+1} - \varepsilon_w \hat{\pi}_t)
\]

\[\iff \] \[\hat{w}_t^* = (1 - \lambda) \hat{w}_t^0 + 1 \left[ (1 - \lambda) \varphi_H \Gamma - (1 - \lambda) \varphi_H \Gamma \frac{1}{1 - \gamma} + \tau \frac{1}{1 - \gamma} \right] E_t \hat{w}_{t+1}
\]

Denote \(\zeta = (1 - \lambda) \varphi_H \Gamma\), and use the above equation to eliminate \(\hat{w}_t^*\) from the average wage equation (equation 2)
\[
\hat{w}_t = (1 - \gamma) (1 - \lambda) \hat{w}_t^0 + (\zeta - \tau \gamma) (\hat{w}_t - E_t \hat{\pi}_{t+1} + \varepsilon_w \hat{\pi}_t)
\]

\[+ (1 - \gamma) \tau (E_t \hat{\pi}_{t+1} - \varepsilon_w \hat{\pi}_t)
\]

\[+ [(1 - \gamma) \zeta - \zeta + \lambda] E_t \hat{w}_{t+1} + \gamma (\hat{w}_{t-1} - \hat{\pi}_t + \varepsilon_w \hat{\pi}_{t-1})\]
\[ [1 - \gamma (\zeta - \iota)] \hat{\bar{w}}_t = (1 - \gamma) (1 - \iota) \hat{\bar{w}}_t^0 - \gamma (\zeta - \iota) (E_t \hat{\bar{\pi}}_{t+1} - \bar{\varepsilon}_w \hat{\bar{\pi}}_t) + (1 - \gamma) \iota (E_t \hat{\bar{\pi}}_{t+1} - \bar{\varepsilon}_w \hat{\bar{\pi}}_t) + [ (1 - \gamma) \zeta - \zeta + \iota] E_t \hat{\bar{w}}_{t+1} + \gamma (\hat{\bar{w}}_{t-1} - \hat{\bar{\pi}}_t + \varepsilon_w \hat{\bar{\pi}}_{t-1}) \]

Finally, after dividing by \([1 - \gamma (\zeta - \iota)]\), the dynamic average wage equation can be expressed as

\[
\Longleftrightarrow \hat{\bar{w}}_t = \lambda_b (\hat{\bar{w}}_{t-1} - \hat{\bar{\pi}}_t + \varepsilon_w \hat{\bar{\pi}}_{t-1}) + \lambda_0 \hat{\bar{w}}_t^0 + \lambda_f E_t (\hat{\bar{w}}_{t+1} + \hat{\bar{\pi}}_{t+1} - \varepsilon_w \hat{\bar{\pi}}_t)
\]

where \(\lambda_b = \dfrac{\gamma}{[1 - \gamma (\zeta - \iota)]}\), \(\lambda_0 = \dfrac{(1 - \gamma) (1 - \iota)}{[1 - \gamma (\zeta - \iota)]}\), and \(\lambda_f = \dfrac{\iota - \gamma \zeta}{[1 - \gamma (\zeta - \iota)]}\),

with \(\zeta = (1 - \iota) \varphi_H \Gamma\), \(\iota = \bar{\beta} (1 - \bar{p}) \gamma\), \(\Gamma = \dfrac{\bar{w}h (1 - \bar{\tau})}{(1 - \iota) \bar{H}}\), \(\varphi_H = \dfrac{(1 - \eta) \bar{H} \beta q^W}{\bar{w}h (1 - \bar{\tau})}\) as previously denoted.