Macroeconomics 1 - lecture notes 4

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1 Idiosyncratic uncertainty

In lecture notes 3, we described a stochastic version of the neoclassical growth model. In that model, uncertainty is related to aggregate productivity and the income of individual households moves one-to-one with aggregate income.

However, the most important economic risks that individuals face are likely to be *idiosyncratic*. For instance, for most people the income uncertainty related to the risk of losing their own job and becoming unemployed is far more important than the uncertainty related to aggregate output fluctuations over a normal business cycle. (However, the idiosyncratic unemployment risk changes over the business cycle, so that aggregate and idiosyncratic risks are correlated in this case.)

Idiosyncratic risks do not matter if financial markets are complete in the sense that individuals can fully insure themselves by trading state contingent assets. For concreteness, assume that the only source of uncertainty in the economy is that every month everyone faces 1 % chance of losing her job and that it always takes one month to find a new job. We can imagine an unemployment insurance scheme that pays everyone one month's salary following a layoff. If the insurance is actuarially fair, then all risk averse individuals would buy such an insurance policy. As a result, individuals' incomes would fluctuate only with aggregate income shocks, not with idiosyncratic risks. We can then often abstract from idiosyncratic uncertainties altogether. This is why macro models that do not feature idiosyncratic uncertainties can often be classified as complete markets models.

Of course, for various reasons (adverse selection, moral hazard, transaction costs,...), financial markets are not complete. Hence, individuals cannot fully insure themselves against all economic risks. Models that are consistent with this observation are often referred to as incomplete markets models. Idiosyncratic uncertainty also means that even if agents are ex-ante identical (have the same preferences and initial endowments), they may end up being in very different economic situations. For instance, the savings of an individual close to retirement depends on its entire labor market history (including unemployment spells). For this reason, models with idiosyncratic uncertainty are also often referred to as heterogenous agent models.

In this lecture note, we present different versions of a life-cycle savings model, where individuals face idiosyncratic income uncertainty. Similar models have often been used to study how different tax-and-transfer schemes influence labour supply and savings incentives over the lifecycle, the distribution of (lifetime) income, and (expected lifetime) welfare. Life-cycle models are especially useful when analysing pension systems. This is because the labour supply incentives created by a pension system are often impossible to capture in a static analysis. (Think of earnings-related pension systems with some earnings test for benefits.)

Here we focus on the individual life-cycle savings and labour supply problem taking prices as given (partial equilibrium analysis). However, we also briefly describe how to extend the life-cycle models to general equilibrium overlapping generations models. For interesting examples of analysis based on similar life-cycle or overlapping generations models see, for instance, Imrohoroglu et al. (1995), Floden and J. Lindé (2001), Conesa et al. (2009), and Blundell et al. (2016).

1.1 Life-cycle savings under income uncertainty

The individual lives at most T periods stating from age $j = 1$. The probability of surviving from age j to age $j+1$, conditional on having survived until age j, is denoted S_j. Each period, the individual makes a (continuous) savings (or consumption) decision. In addition to lifetime uncertainty, the individual also faces income uncertainty in the form of an income shock s that follows a discrete first-order Markov-process.

In its recursive form, the individual problem read as:

$$V_j(a_j, s_j) = \max_{c_j}\{u(c_j) + \beta S_j E_j V_{j+1}(a_{j+1}, s_{j+1})\} \tag{1}$$

subject to

$$a_{j+1} = (1+r)a_j + s_j w_j + b_j - c_j \tag{2}$$

$$a_{j+1} \geq 0 \tag{3}$$

Here w_j denotes (after-tax) wage rate, which may depend on age, and b_j is an age-dependent lump-sum transfer that can be used to capture e.g. a flat-rate pension benefit.

The expectation is taken over the next period income shock s_{j+1} given current period shock s_j. Assume, for example, that s may take two values, namely s^1 and s^2. Then $E_j V_{j+1}(a_{j+1}, s_{j+1})$ can be expressed as $prob(s_{j+1} = s^1|s_j)V_{j+1}(a_{j+1}, s^1) + prob(s_{j+1} = s^2|s_j)V_{j+1}(a_{j+1}, s^2)$.

In order to analyse the model, one first needs to solve the above individual problem. That means finding the value functions and the associated policy functions $a_{j+1}(j, a_j, s_j)$ that solve the above Bellman equation. The next step is usually to simulate a large number of individual labour supply and savings paths for different realizations of the income shock process. *lifecycle.m* shows how to solve and simulate the above problem for a two-state income process.

1.2 Life-cycle savings and labour supply

We now extend the above model by introducing a tradeoff between consumption (of the consumption good) and leisure. The individual problem is:

$$V_j(a_j, s_j) = \max_{a_{j+1}, n_j} \{u(c_j, 1 - n_j) + \beta S_j E_j V_{j+1}(a_{j+1}, s_{j+1})\} \tag{4}$$

subject to

$$a_{j+1} = (1+r)a_j + s_j w_j n_j + b_j - c_j \tag{5}$$

$$a' \geq 0 \tag{6}$$

$$0 \leq n_j < 1 \tag{7}$$

Having two continous decision variables (here savings and labour supply) may increase the computational substantially compared to a case with just one continous decision variable. Notice, however, that in this case the labour supply decision is static in nature. Given next period savings, one can solve for the optimal labour supply decision using a static first-order condition. Depending on the utility function, one may then be able to derive an analytical expression for the optimal labour supply given savings, which would reduce the computational burden a lot.

1.3 Earnings related pension system

Life-cycle models are often used to analyse the labour-supply incentives of relatively complex tax-and-transfer schemes such as earnings related pension system, where pension benefits depend on individual's earnings during working life. However, modelling such systems accurately usually requires introducing additional state variables to the individual problem.

As an example, let us consider a pension system where each euro earned in age j contributes κ_j euros to future pension benefits that can be withdrawn after some specific "retirement age". Presumably, the individual labour supply and savings decisions close to the retirement age depend on accrued pension rights. Here we consider a discrete labour supply decision.

The individual problem reads as:

$$V_j(a_j, e_j, s_j) = \max_{a_{j+1}, n_j} \{u(c_j, 1 - n_j) + \beta S_j E V_{j+1}(a_{j+1}, e_{j+1}, s_{j+1})\} \tag{8}$$

subject to

$$a_{j+1} = (1+r)a_j + s_j w_j n_j + b_j - T(j, b_j, s_j w_j n_j) - c_j \tag{9}$$

$$b_j = B(j, e_j, s_j w_j n_j) \tag{10}$$

$$e_{j+1} = e_j + \kappa_j s_j w_j n_j \tag{11}$$

$$a' \geq 0 \tag{12}$$

$$n_j \in n^1, n^2, ..., n^I \tag{13}$$

Here $T(j, b_j, s_j w_j n_j)$ is a function that determines the periodic tax bill of the individual. In this formulation, taxes may depend on age, accrued pension rights, and current wage income. Function $B(j, e_j, s_j w_j n_j)$ in turn determines transfers, including the pension benefit. For instance, it might be speficied so that $B(j, e_j, s_j w_j n_j) = e_j$ for $j \geq j_r$ and $B(j, e_j, s_j w_j n_j) = 0$ for $j < j_r$, where j_r is the retirement age (or the eligibility age for pension benefits). It could also include e.g. a flat-rate pension benefit and unemployment insurance.

More generally, T and B could be arbitrary functions of the current state and decision variables as well as the prices and parameters that the individual takes as given. Hence, this formulation already allows modelling relatively complex tax-and-transfer schemes.

1.4 Exogenously vs. endogenously incomplete markets

It is perhaps worth noting that here the market incompleteness itself is exogenous: it is just assumed that only self-insurance via riskless capital holdings is available. There is also an important strand of literature that studies *endogenously* incomplete markets. In those models, the reason for markets being incomplete relates to the lack of commitment. Typically, it is assumed that agents may always walk away from a risk-sharing arrangement at the cost of being excluded from all risk sharing in the future. For details and references, see e.g. the textbook by Ljungqvist and Sargent.

1.5 General equilibrium OLG models

The life cycle models described above can be extended to general equilibrium overlapping generations (OLG) models, where every period a new cohort is born and individuals of ages 1 to T are living at the same time. Having a very large number of individuals (technically, a continuum of individuals) implies that despite idiosyncratic uncertainty, there is no aggregate uncertainty. With constant survival probabilities and a constant mass of "newborn" individuals, the model features a steady state where the distribution over individuals over age and other state variables is constant over time. One could then introduce similar competive capital and labour markets as in lecture notes 2.

An OLG model would also allow accounting for the government budget constraint that includes the pension system. The pension system could be fully funded, pay-as-you-go, or a mixture of the two. In a pay-as-you-go system, the pension benefits are financed by taxing the current workers. In a fully funded system, the pension contributions paid by each cohort are invested in the capital markets until the members of the cohort start withdrawing pension benefits.

In addition to the steady analysis, the OLG models can be used to consider the transitionary dynamics following e.g. a pension or tax reform. In the absence of aggregate uncertainty, aggregate dynamics would still be deterministic.

References

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