Macroeconomics 1 - lecture notes 2

Niku Määttänen

October 2, 2018

1 Introducing distortionary taxes into the neoclassical growth model

This second lecture note introduces distortionary taxation into the neoclassical growth model. We consider a model with government bonds. Having government bonds allows smoothing tax rates over time. We also explain briefly how to "calibrate" the model, and illustrate how to use the model for policy analysis. Because of distortions, the competitive equilibrium and the solution of the social planner's problem no longer coincide. We therefore need to solve directly for the competitive equilibrium.

1.1 The household problem

There is a government that finances public expenditures with flat-rate taxes. Instead of assuming that the government budget needs to be balanced every period, we let the government borrow or lend from the private sector using one-period bonds denoted by b.

The problem of the household in period 1 is

$$\max_{\{c_t,b_{t+1},k_{t+1},n_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, 1-n_t) \tag{1}$$

subject to the sequence of periodic budget constraints and two no-Ponzi-game conditions:

$$(1+\tau_t^c)\,c_t + k_{t+1} + \frac{b_{t+1}}{R_t} = \left(1+(1-\tau_t^k)r_t\right)k_t + (1-\tau_t^n)w_t n_t + b_t \tag{2}$$

$$k_1 > 0 \text{ and } b_1 \text{ given} \tag{3}$$

$$\lim_{t->\infty}\prod_{s=2}^{t}\frac{1}{1+(1-t_s^k)r_s}k_{t+1} = 0 \tag{4}$$

$$\lim_{t->\infty}\prod_{s=1}^{t}R_s^{-1}b_{t+1} = 0. \tag{5}$$

Here $1/R_t$ is the price in period t of a one-period government bond that pays one unit of the consumption good in period $t+1$ and τ^c, τ^k and τ^n are the tax rates on consumption, return to business capital and labor, respectively.

The household has here two savings vehicles: capital goods and government bonds. Since the model is deterministic, it is clear that the assets must yield the same return in equilibrium. It is easy to show that this implies the following:

$$R_t = 1 + (1-\tau_{t+1}^k)r_{t+1}.$$

We have assumed that the government bond is tax-free. This is without loss of generality. Taxing bonds would just lower their price i.e. increase the interest rate that the government needs to pay.

1.2 Deriving the present value budget constraint

Since capital and goverment bonds yield the same return, the household's optimal policy is not fully determined. It is therefore helpful to eliminate one of the two assets. We eliminate government bonds from the household problem by replacing the periodic household budget constraint with a present value budget constraint.

Using the periodic budget constraint for period 2 and solving for b_2 gives

$$b_2 = (1+\tau_2^c)\,c_2 + k_3 + \frac{b_3}{R_2} - \left(1+(1-\tau_2^k)r_2\right)k_2 - (1-\tau_2^n)w_2 n_2.$$

Plugging this expression into period 1 budget constraint gives

$$(1+\tau_1^c)\,c_1 + k_2 + \frac{(1+\tau_2^c)\,c_2}{R_1} + \frac{k_3}{R_1} + \frac{b_3}{R_1 R_2} = \frac{R_2^k k_2}{R_1} + \frac{(1-\tau_2^n)w_2 n_2}{R_1} + R_1^k k_1 + (1-\tau_1^n)w_1 n_1 + b_1$$

where $R_t^k = 1 + (1 - \tau_t^k) r_t$.

By eliminating in the same manner b_3, b_4 and so on, we end up with

$$\sum_{t=1}^{\infty} \left(\prod_{s=1}^{t-1} R_s^{-1} \right) \left[\left(1 + (1 - \tau_t^k) r_t\right) k_t + (1 - \tau_t^n) w_t n_t - (1 + \tau_t^c)\, c_t - k_{t+1} \right] + b_1 = 0$$

where we have imposed the condition that

$$\lim_{t \to \infty} \left(\prod_{s=1}^{t-1} R_s^{-1} \right) \frac{b_{t+1}}{R_t} = 0.$$

Following lecture notes 1, it can be shown that the transversality condition with respect to b_t implies this condition. As a result, also the no-Ponzi-game condition holds. Defining $p_t = \left(\prod_{s=1}^{t-1} R_s^{-1} \right)$, and assuming that $b_1 = 0$, we can now write the present value budget constraint of the household as

$$\sum_{t=1}^{\infty} p_t [(1 + (1 - \tau_t^k) r_t) k_t + (1 - \tau_t^n) w_t n_t - (1 + \tau_t^c)\, c_t - k_{t+1}] = 0 \tag{6}$$

Again, p_t may also be defined as the before-tax price of period t consumption in terms of period 1 consumption.

We have now derived a present value budget constraint starting from the periodic budget constraints. In the process, we also eliminated government bonds from the household problem. It is also possible to eliminate the capital stock from the present value budget constraint (see section 3.5 in Lecture notes 1). However, the above form of the household budget constraint is often used.

1.3 Household optimality conditions

The first-order conditions characterizing individually optimal behavior may be written as

$$u_{c_t} (1 - \tau_t^n) w_t = u_{l_t} (1 + \tau_t^c) \tag{7}$$

$$\frac{u_{c_t}}{1 + \tau_t^c} = \frac{\beta u_{c_{t+1}}}{1 + \tau_{t+1}^c} (1 + (1 - \tau_{t+1}^k) r_{t+1}) \tag{8}$$

In addition to these constraints, the solution must satisfy the present value budget constraint.

These first-order conditions already tell us something about how the different tax rates should influence labour supply and savings decisions. The first-order condition related to labour supply reveals that labor income and consumption taxes influence the labour supply decision in a very similar way. A higher consumption tax has the same effect as a higher labour income tax. Similarly, the Euler equation tells us that regarding savings incentives, an increasing path for consumption taxes is equivalent to a strictly positive capital income tax.

1.4 Firm's problem

The firm's problem is the same as before, so we have

$$
\begin{aligned}
r_t &= f_{k_t} - \delta && (9) \\
w_t &= f_{n_t}, && (10)
\end{aligned}
$$

1.5 Government budget constraint

The government's periodic budget constraint can be written as

$$\tau_t^c c_t + \tau_t^k r_t k_t + \tau_t^n n_t w_t + \frac{b_{t+1}}{R_t} - b_t - g_t = 0 \tag{11}$$

and its present value budget as

$$\sum_{t=1}^{\infty} p_t \left(\tau_t^c c_t + \tau_t^k r_t k_t + \tau_t^n n_t w_t - g_t \right) = 0 \tag{12}$$

A budget-feasible government policy is a sequence of tax rates $\{\tau_t^k, \tau_t^n, \tau_t^c\}_{t=1}^{\infty}$ and government expenditures $\{g_t\}_{t=1}^{\infty}$ that satisfy (12). (More generally, the government budget constraint may be written as an inequality constraint. However, we are usually only interested in government policies that satisfy the government budget constraint with equality since there is no reason to collect more taxes than what is necessary to finance the government spending.)

1.6 Aggregate consistency

Aggregate resource constraint reads as:

$$c_t + g_t + k_{t+1} = f(k_t, n_t) + (1 - \delta)k_t.$$

1.7 Competitive equilibrium

For a given initial capital stock, k_1, initial government bonds, b_1, and a budget-feasible government policy $\{\tau_t^k, \tau_t^n, \tau_t^c, g_t\}_{t=1}^{\infty}$, a competitive equilibrium consists of prices $\{p_t, r_t, w_t\}_{t=1}^{\infty}$ and allocations $\{c_t, n_t, k_{t+1}, b_{t+1}\}_{t=1}^{\infty}$ such that *i*) given prices and tax rates, the allocation solves the household problem, *ii*) the allocation satisfies the aggregate resource constraints, *iii*) factor returns r_t and w_t are given by equations (9) and (10) and *iv*) government bonds are determined (as a residual) from the government's periodic budget constraints (11).

1.8 Solving for the competitive equilibrium

We typically specify the tax system so that at least one of the tax rates is allowed to be adjusted to balance the government's intertemporal budget constraint (instead of taking the whole tax system as given). The competitive equilibrium can then be found by solving a system of non-linear equations that consists of the household and firm optimality conditions together with two of the following constraints: the aggregate resource constraint, the government present value budget constrain, and the household present value budget constraint. Again, we need to assume that the economy converges to a steady state by some period T.

When solving the model, it is useful to note that we can write the government present value budget constraint, for instance, as follows:

$$\begin{aligned}\sum_{t=1}^{\infty} p_t\left(\tau_t^c c_t + \tau_t^k r_t k_t + \tau_t^n n_t w_t - g_t\right) &= \sum_{t=1}^{T} p_t\left(\tau_t^c c_t + \tau_t^k r_t k_t + \tau_t^n n_t w_t - g_t\right) \\ &+ p_T \frac{1}{(1-\tau^k)r}\left(\tau^c c + \tau^k r k + \tau^n n w - g\right),\end{aligned}$$

where variables without time index refer to steady state values. The second term in the right-hand side is the present value of government surpluses (or deficits) from period $T+1$ to infinity.

1.9 Calibrating the model

"Calibration" refers to the process of choosing parameter values so that the model is in some sense realistic. A straigthforward way of calibrating this type of dynamic general equilibrium model is to set some parameter values “exogenously” based on empirical observations or estimates and other parameter values “endogenously” so that the steady state of the model matches some relevant empirical statistics.

For instance, we may want to use this model to analyze potential tax reforms. In order to solve the model numerically, we need to determine all the functional forms and parameter values, including the pre-reform steady state tax rates. In addition, we need to specify the length of the model period.

For concreteness, let us assume constant-relative-risk-aversion (CRRA) utility function $u(c, 1-n) = (c^{\gamma}(1-n)^{1-\gamma})^{1-\sigma}$ and Cobb-Douglas production function $f(k,n) = k^{\alpha}l^{1-\alpha}$. Then the parameter values that need to be specified are $\delta, \beta, \gamma, \sigma, \alpha, \tau^k, \tau^c, \tau^n$. We also need to make some assumptions regarding g and b in the initial steady state and the steady state government deficit or surplus. This is because the same tax revenue can be used to finance either gov. spending or interest payments on government debt (if $b > 0$).

It would then be natural to choose the initial (steady state) tax rates exogenously so that they reflect the current tax structure e.g. by setting them equal to some estimates of the average effective tax rates on consumption, labor and capital income (in a given country). It is also common to set the capital depreciation rate based on empirical estimates. Notice that the proper depreciation rate depends on the model period, so the model period has to be specified first.

It is hard to find relatively precise estimates of the preference parameters β, γ, σ. A standard approach here is to fix σ at a “reasonable” value and to choose β and γ, as well as α, “endogenously” so as to replicate certain empirical aggregate ratios in the steady state of the model. Specifically, we might want to set these parameters so as to match the capital-to-output ratio $k/f(k,n)$ (often estimated to be around 3 for annual ouput for the US economy), the share of available time used for working (perhaps around 1/3) and the capital income share (around 40% or so). One can then vary σ to see how it affects the results. (Notice that σ doesn't affect the steady state.)

1.10 Evaluating welfare effects

We often want to quantify the welfare gains or losses associated with a tax reform. Typically, this is done by computing the relative increase in consumption that would make the households indifferent between the initial steady state, or "the status quo", and the tax reform.

In other words, we find a number x such that

$$\sum_{t=1}^{\infty} \beta^{t-1} u(c_t^r, 1 - n_t^r) = \sum_{t=1}^{\infty} \beta^{t-1} u((1+x)c^{sq}, 1 - n^{sq}) = \frac{u((1+x)c^{sq}, 1 - n^{sq})}{1 - \beta}$$

where c^{sq} and n^{sq} denote consumption and labor in the status quo steady state and c_t^r and n_t^r denote consumption and labor supply in the competitive equilibrium that is associated with the tax reform. The interpretation of $x = 0.1$, for instance, would be that compared to the status quo, the tax reform in question increases welfare by 10% in terms of "equivalent consumption compensation".