Should Old-age Benefits Be Earnings-tested?*

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Abstract

We study the welfare effects of earnings testing flat-rate old-age benefits in a quantitative overlapping generations model with idiosyncratic labor income risk. In our model economy, even a moderate earnings testing reduces individuals' expected lifetime utility, whenever other taxes are taken into account. Moreover, it also lowers the realized lifetime utilities of those with lowest realized lifetime utilities.

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Keywords: Social security; Retirement; Earnings testing

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1 Introduction

Private insurance agencies usually cannot cover for risks whose realization is revealed before children become adults. For example, there is no insurance market for innate talent, social status of one's parents, or the quality of education and peer groups one enters. This renders a significant part of lifetime income risks uninsurable from the perspective of a hypothetical citizen behind a veil of ignorance. Furthermore, private information implies in many cases that an insurance provider would suffer from adverse selection problems even when covering for risks realized during working life. In particular, it is difficult for individuals to insure themselves against labor income uncertainty. A government, however, can substitute for the missing private insurance markets by mandatory social insurance (see Friedman 1953, Harsanyi 1953, Rawls 1971). This provides an efficiency rationale for redistributive social insurance, a central part of which is in most countries provided through old-age benefits.

Even if there is a consensus to organize a redistributive social security, important decisions remain. One of the most important ones is whether benefits should be paid to everyone above a certain age or only to those with no or low wage income, i.e. whether benefits should be earnings-tested or not. Earnings testing old-age benefits imposes an implicit tax on continued work after entitlement age. Such implicit tax comes on top of wage taxes. Gruber and Wise (1999) provide thorough empirical evidence suggesting that such taxes are common in OECD countries and that the implicit tax burden plays an important role in retirement decisions.

The stated aim of earnings testing is to target benefits to the poor. When benefits are earnings-tested, those with little wage income after the entitlement age receive higher benefits than others. Earnings testing may improve the risk sharing properties of social security if earnings-tested benefits also imply more redistribution from individuals with high lifetime earnings to those with low lifetime labor income than uniform benefits. This is à priori unclear, however. For instance, it may be that it is mainly the relatively wealthy individuals who benefit from earnings testing because they can afford to retire early.

In this paper, we analyze the welfare effects of earnings testing old-age benefits within a calibrated, general equilibrium overlapping generations model with endogenous labor supply and retirement behavior. In policy experiments, we keep the size of the social security system constant, relative to the total labor income in the economy. Therefore, money saved by earnings-testing benefits is used to increase the size of full benefits. By fixing the size of the PAYG social security system, we also separate intragenerational redistribution, which would be the purpose of an earnings test, from intergenerational redistribution that any PAYG social security system implements whenever the interest rate does not equal the growth rate of the economy.

We take into account both permanent labor productivity differences and transitory labor productivity shocks, calibrating the productivity process to the US data. Permanent productivity differences may be thought of as differences in innate ability. They capture the stylized fact that people enter the labor market with different skills, receiving different hourly wages. Transitory shocks, on the other hand, are modelled as a random walk that affects the development of earnings opportunities, in addition to underlying innate productivity and a common life cycle wage profile. Financial markets are incomplete in that individuals cannot insure themselves against the labor income risk and cannot borrow against future labor income.

We take as our starting point a pay-as-you-go (PAYG) social security system with flat-rate benefits and without earnings-testing. We then analyze what would be the effect of introducing an earnings test. We study both an earnings test in which all earned income reduces benefits proportionally and an earnings test in which sufficiently low earnings do not reduce benefits.

We consider both the expected lifetime utility of individuals with different innate abilities and the distribution of realized lifetime utilities. These two approaches provide complementary criteria for ethical evaluation and political decision-making. Considering the expected lifetime utilities would be highlighted from a utilitarian perspective. Comparing ex post outcomes may be more relevant if the social security system is designed to reduce welfare inequality. From the Rawlsian perspective, of prime interest would be the effect of earnings-testing old-age benefits on the welfare of the individuals with lowest lifetime utility. Clearly, a given social security system may increase the lot of the most unlucky individuals even if it reduces individuals' expected lifetime utility at birth.

Our results suggest that earnings testing flat-rate benefits is unlikely to work as it is intended. In our model economy, even a moderate earnings test not just reduces individuals' expected lifetime utility but also the realized lifetime utility of almost all individuals, including those with the lowest realized lifetime utilities.

There are two reasons for why earnings testing does not improve social insurance, according to our analysis. First, the aggregate labor supply elasticity of individuals who are old enough to receive old age benefits is quite high. As a result, even a moderate increase in the effective taxation of labor at older ages results in a relatively large decrease in aggregate labor supply. Such a strong labor supply response implies that the unreduced full benefit cannot be increased much, and that the general taxation needs to be increased to collect enough revenue to finance other public expenditures.

Second, while the correlation between lifetime earnings and earnings after the entitlement age is positive, it is not very strong. As a result, the earnings test, which taxes earnings after the entitlement age, does not redistribute in a very systematic way from the individuals who have high average labor productivity to those who have low average labor productivity. One reason individuals with high lifetime earnings often retire earlier than individuals with lower lifetime earnings is the interaction of individual savings and labor supply decisions under income uncertainty: A young individual who is hit by positive income shocks works a lot and saves a large fraction of his earnings. However, even if the individual continues to be hit by positive shocks, at some point he will start to use his savings to consume more leisure. In other words, those who turn out to have high labor productivity for most of their lifetime are also likely to be wealthy at old age and that makes them retire relatively early.

Previous analyses of the optimality of earnings testing of social security have mainly focused on disability risks. In pioneering contributions, Diamond and Mirrlees (1978, 1986) conclude that optimal benefits are structured so that the healthy are indifferent as to whether to mimic the disabled or continue working. More recently, Golosov and Tsyvinski (2006) suggest using asset testing to deter falsely claimed disability benefits. Cremer et al. (2004) conclude that if high-productivity individuals also have lower utility cost of working, then earnings testing can be used to deter high-productivity types from pooling with low-productivity types. Two main features separate our paper from this strand of literature. First, in these papers, disability (or higher cost of working) is always modelled as a permanent state. In contrast, we ask whether old-age benefits should be earnings-tested in the presence of income shocks that are not permanent. Second, we use a more detailed model with the aim of providing a quantitatively credible assessment of the trade-offs involved.

There are, of course, many papers analyzing the risk sharing aspects of a social security system using similar overlapping generations models with idiosyncratic labor income uncertainty. Examples include İmrohoroğlu et al. (1995), Conesa and Krueger (1999), and Gomes and Michaelides (2003). However, most of this literature focuses on a PAYG social security system versus a fully funded system.[1] One exception is provided by Fehr and Habermann (2008) who study the optimal relation between benefits and lifetime earnings within the German earnings-related pension system. They find that because of risk sharing, the link between benefits and lifetime earnings should be relatively progressive. Sefton et al. (2008) consider changes in the degree of means testing, which takes also individual savings into account, within the UK state pension system. They find that a moderate means testing increases households' expected lifetime utility at birth. As the authors explain, this is related to the fact that means testing is also a way to (indirectly) tax lifetime earnings, since savings and lifetime earnings are positively correlated.

Apart from the different policy question, our model and methodology are quite different from both of these studies. Fehr and Habermann (2008) force agents in their model to retire at a fixed retirement age. For our analysis, modelling the retirement decision is of course crucial. Sefton et al. do not impose a fixed retirement age but model labor supply as a dichotomous choice in every period, whereas we model both the extensive and intensive margin. Sefton et al. (2008) use household data and calibrate their model so as to match many distributional features. On the other hand, as far as the welfare results are concerned, they do not provide a sensitivity analysis. We follow a much simpler calibration strategy but complement our analysis with a systematic sensitivity analysis regarding the preference

[1] There is an important and related strand of literature that focuses on how social security systems influence individuals' retirement decisions using similar models of labor supply. Earlier examples of this literature include Gustman and Steinmeier (1986), Rust and Phelan (1997), and Stock and Wise (1990) and a recent example is French (2005). These analyses model existing social security systems in great detail and often also estimate many of the model parameters using household or individual level data. However, they do not account for the welfare effects of alternative social security systems.

parameters.

The paper is organized as follows. In the next section, we present our model economy. In the third section, we explain the calibration. Results are presented in section 4. Section 5 presents sensitivity analysis, and section 6 concludes. The appendix explains some computational issues.

2 Model

The analysis is based on a discrete time overlapping generations model where individuals decide upon consumption, savings, and labor supply. All individuals live from age 1 until age J. We use j to denote individual's age. Individuals face uncertainty about their future wage levels. Financial markets are incomplete in the sense that individuals cannot privately insure themselves against labor income uncertainty. Hence, a social security system, or some other redistribution scheme, may improve welfare by providing a partial substitute for the missing private insurance market. The interest rate and the wage rate are determined competitively. The parameters of the social security system are chosen so that the budget of the system is always balanced.

We consider only steady states where all prices, tax and transfer parameters, the demographic structure, and the distribution of individuals over their states are constant over time. This restriction helps us in focusing on the possible trade-off between providing insurance against individual uncertainty in the labor market and minimizing the distortionary effects that the social security system has on labor supply.

2.1 Firms

A representative firm employs business capital, K, and effective labor, L, to produce output goods, Y. The output good may be costlessly converted into consumption goods or capital. The production function is

$$Y = AK^{\alpha}L^{1-\alpha}. \tag{1}$$

The firm's first-order conditions for profit maximization imply that the interest rate, r, and the wage rate per unit of effective labor, w, equal the marginal productivities of business capital and effective labor. That is,

$$r = \alpha A K^{\alpha-1} L^{1-\alpha} - \delta \tag{2}$$

and

$$w = (1-\alpha) A K^{\alpha} L^{-\alpha}, \tag{3}$$

where δ is the depreciation rate of business capital and A is total factor productivity.

2.2 Labor productivity

Individuals have different labor productivities. The labor productivity of an individual of age j is

$$w_j = w h_j e^{v+z_j}, \tag{4}$$

where h_j denotes the average, age-specific labor productivity component of individuals of age j, v denotes individual's permanent productivity shock, and z_j a temporary shock.[2] The temporary shock follows an AR(1) process:

$$z_j = \eta z_{j-1} + \varepsilon, \tag{5}$$

where η determines the degree of persistence and ε is independently, identically and normally distributed with mean zero and variance σ_ε^2.

2.3 The social security system

There is a government that collects taxes in order to pay for government expenditures, G, and to run a PAYG social security system. Government expenditures are financed by a proportional tax on labor and capital income, τ, and the social security system by a proportional tax, θ, on labor income alone.

Old-age benefits depend on individual's current labor income and age and we denote them by $B(w_j l_j, j)$. We assume the following benefit rule.

$$B(w_j l, j) = \{ \begin{array}{c} 0 \text{ if } j < j_r \\ \max[s - \lambda(1-\tau-\theta)\max[w_j l - f, 0], 0], \text{ if } j \geq j_r \end{array} . \tag{6}$$

[2] This specification for the labor productivity follows Floden and Lindé (2001).

Individuals are entitled to benefits from age j_r onwards. The parameter $0 \leq \lambda \leq 1$ determines the implicit tax rate of the earnings test and the parameter $f \geq 0$ denotes an exemption (free income that may be earned without effects on old-age benefits). After-tax labor income below the exemption does not reduce benefits. When $\lambda = 0$, there is no earnings test and all individuals above the retirement age receive the same full benefit, s. When $\lambda = 1$, benefits are fully earnings-tested for earnings above the exemption level. In that case, if there is no exemption, an individual with a net income higher than s receives no benefits. This means that the social security system implies a 100% effective marginal tax rate on labor whenever net labor income is less than s and earnings are above the exemption level.

A higher λ should decrease average labor supply among the elderly. On the other hand, by reducing the benefits of individuals with high labor income, a higher λ may also imply a higher full benefit, s, via the social security budget constraint. As discussed in the introduction, this may help to target larger benefits to individuals who would otherwise have very low consumption possibilities. For a given $\lambda > 0$, a higher exemption, f, in turn should imply a lower full benefit. However, an exemption may also help to target benefits for low income individuals, because a larger part of their income is not taken into account when earnings testing the benefit. Clearly, if $\lambda = 0$, the exemption does not matter and as the exemption becomes increasingly large, earnings testing is effectively eliminated no matter what is λ.

In our analysis, we keep the contribution rate, θ, fixed and vary the parameters in the benefit rule. Hence, the size of the pension system is fixed relative to the total wage bill.[3] Given the contribution rate, the implicit tax rate, and the exemption, the full benefit, s, is then determined so that the budget of the social security system (defined below) is balanced.

2.4 Individual's problem

Each period, individuals have a time endowment of one. Their decision variables are consumption, c, labor supply, l, and savings, k. Periodic utility is given by $u(c, l)$. The individual discount factor between periods is β. The individual state variables are current savings, k, the temporary productivity shock, z, the permanent productivity shock, v, and age, j. In its

[3]We also considered fixing the absolute value of all pension benefits. We will discuss the results of that experiment at the end of section 4.2.

recursive form, the problem of an individual of age j reads as:

$$V_j(k_j, z_j, v) = \max_{k_{j+1}, l_j} \{u(c_j, l_j) + E_{z_{j+1}|z_j} \beta V_{j+1}(k_{j+1}, z_{j+1,}, v)\} \tag{7}$$

subject to

$$
\begin{aligned}
0 &\leq l_j \leq 1 && (8)\\
c_j + k_{j+1} &= [1 + (1-\tau)r]k_j + (1-\theta-\tau)w_j l_j + B(w_j l_j, j) && (9)\\
w_j &= w h_j e^{v+z_j} && (10)\\
z_{j+1} &= \eta z_j + \varepsilon && (11)\\
k_{j+1} &\geq 0. && (12)
\end{aligned}
$$

The first constraint is the time constraint. The second one is the flow budget constraint: consumption and tomorrow's savings equal the sum of current savings, net interest income, net labor income, and old-age benefits. The third equation defines the wage rate per units of time spent working. The fourth equation describes the evolution of labor productivity and the fifth equation states the borrowing constraint.

We assume that individuals are born without initial assets. That is, $k_1 = 0$.

2.5 Competitive equilibrium

Let Φ denote a joint measure over individual states. Given the contribution rate, θ, parameters λ and f of the benefit rule, and government expenditures, G, the steady state competitive equilibrium consists of individual decision rules $c_j(k, z, v)$, $k_j(k, z, v)$, $l_j(k, z, v)$, a value function $V_j(k, z, v)$, aggregate quantities K and L, prices r and w, tax rate, τ, and a measure Φ describing the distribution of individuals such that:

1) Individual policies and the value function solve the individual's problem.

2) Capital stock, aggregate effective labor, and aggregate production are given by:

$$
\begin{aligned}
K &= \int k_j(k, z, v)\Phi(dk \times dz \times dv \times dj), && (13)\\
L &= \int h_j e^{v+z_j} l_j(k, z, v)\Phi(dk \times dz \times dv \times dj) && (14)\\
Y &= AK^{\alpha}L^{1-\alpha} && (15)
\end{aligned}
$$

3) Prices are determined by (2) and (3).

4) The budget of the social security system is balanced:

$$\int \theta w_j l_j(k,z,v)\Phi(dk \times dz \times dv \times dj) = \int B(w_j l_j(k,z,v), j)\Phi(dk \times dz \times dv \times dj). \quad (16)$$

5) Government tax revenue equals government expenditures:

$$\int \tau[w_j l_j(k,z,v) + rk_j(k,z,v)]\Phi(dk \times dz \times dv \times dj) = G. \quad (17)$$

6) The sum of individual consumption, investments and public consumption equals total production:

$$\int c_j(k,z,v)\Phi(dk \times dz \times dv \times dj) + \delta K + G = Y. \quad (18)$$

7) The distribution, Φ, follows from individual decision rules and the labor productivity process.

3 Calibration

A model period corresponds to one year. We assume that individuals enter the model economy at real age 20 and live 66 periods, the last period corresponding to real age 85. The aggregate mass of all individuals is normalized to one.

We assume that intratemporal preferences over consumption and leisure are defined by a constant elasticity of substitution function and that the agents have a constant relative risk aversion. We also assume that there is a fixed cost of work to take into account the time required for commuting and other work-related routines. The fixed cost can explain the empirical fact that individuals typically either spend a substantial fraction of their annual time endowment at work or do not work at all (French, 2005). The intuition is that the fixed cost of working generates reservation wages below which individuals do not participate in the labor market. Just above a reservation wage, individuals may spend a substantial fraction of their time endowment at work.

The periodic utility function $u(c, l)$ is

$$
\begin{aligned}
\frac{[(\mu c^{\rho}+(1-\mu)(1-l-\phi I(l))^{\rho})^{\frac{1}{\rho}}]^{1-\sigma}}{1-\sigma}, \text{ for } \rho &\neq 0, \sigma \neq 1 \\
\frac{[c^{\mu}(1-l-\phi I(l))^{1-\mu}]^{1-\sigma}}{1-\sigma}, \text{for } \rho &= 0, \sigma \neq 1 \\
\frac{1}{\rho}\log[\mu c^{\rho}+(1-\mu)(1-l-\phi I(l))^{\rho}], \text{for } \rho &\neq 0, \sigma = 1 \\
\mu \log c+(1-\mu)\log(1-l-\phi I(l)), \text{for } \rho &= 0, \sigma = 1
\end{aligned}
\tag{19}
$$

Here $\mu > 0$ determines the weight of consumption relative to leisure, $\sigma > 0$ the risk aversion, and $\phi \geq 0$ the fixed cost of work measured in time. The parameter $\rho < 1$ determines the elasticity of substitution between consumption and leisure, $\frac{1}{1-\rho}$. The indicator function $I(l)$ takes the value of zero for $l = 0$ and one for $l > 0$.

In the benchmark case, the risk aversion parameter is set at $\sigma = 2$, a relatively conventional value. The macroeconomic literature tends to use a Cobb-Douglas specification which corresponds to $\rho = 0$. We take this to be our benchmark. We set the fixed cost parameter, somewhat arbitrarily, at $\phi = 0.10$. We vary both the elasticity parameter, ρ, and the fixed cost of work parameter in our sensitivity analysis. The aggregate labor supply elasticity depends on the (endogenous) distribution of reservation wages.

We take the calibration of the wage process from Floden and Lindé (2001). They estimate an AR(1) process for the relative wage level both for the US and for Sweden. We use their estimates for the US which are based on data from the Panel Study of Income Dynamics. The process for the temporary shock is fully characterized by the persistence parameter ρ and the variance σ_{ε}^2. The values are $\rho = 0.9136$ and $\sigma_{\varepsilon}^2 = 0.0426$. Assuming that the permanent shock follows a normal distribution with mean zero, they estimate its variance to be $\sigma_{\upsilon}^2 = 0.1175$.

When solving and simulating the model, we treat the temporary productivity shock, z, as a continuous variable, but approximate the normal distribution of ε with a discrete process that can take 5 values.[4] Similarly, we consider just two levels for the permanent shock, υ. Individuals with a high permanent shock, which we will refer to as 'high-ability types', are on average about twice as productive as individuals with a low permanent shock or 'low-ability types'. We assume that for all individuals $z_1 = 0$. This means that the labor productivity distribution gets more and more dispersed among older individuals.

[4]Specifically, ε gets values $\{-0.61, -0.31, 0, 0.31, 0.61\}$ with probabilities $\{0.012, 0.21, 0.55, 0.21, 0.012\}$.

The age-profile of labor productivity, $\{h_j\}_{j=1}^{J}$, is taken from table 2 in Floden and Lindé (2001). Their estimates are as follows

$$\log(wage) = -3.330 + 0.076age - 0.00077age^2, \tag{20}$$

where 'age' denotes real age in years. This gives a usual hump-shaped pattern for average labor productivity. We further normalize the profile so that the average labor productivity is equal to one.

We set the technology parameters at $\delta = 0.06$ and $\alpha = 0.333$, which are both relatively standard values. We calibrate the social security system so that the size of the system is comparable to the US social security system. In particular, we set the entitlement age, j_r, so that it corresponds to real age 62, which is the entitlement age in the US, and the contribution rate at $\theta = 0.153$, which is the actual contribution rate.[5] The contribution rate pins down the full benefit, s, via the social security budget constraint (16). For the benchmark calibration, we set $\lambda = 0$ so that there is no earnings testing.

We are left with the following parameters: $\{\beta, \mu, G, \tau\}$. We choose them so as to match the following targets: 1) capital-to-output ratio $K/(AK^{\alpha}L^{1-\alpha}) = 3.0$, 2) government expenditures-to-output ratio $G/(AK^{\alpha}L^{1-\alpha}) = 0.20$, 3) a participation rate at the age of 62 equal to 0.70. The first two targets are relatively standard and are based on national income and product accounts. The second target pins down both public expenditures and the income tax rate via the government budget constraint (17). An employment rate of 0.70 corresponds roughly to the employment rate of healthy individuals of age 62 in the PSID data as reported in French (2005). We match the employment rate at the entitlement age because it is crucial that the model features realistic aggregate labor supply behavior around that age. The reason we do not match the employment rate for older ages is that it is likely to be strongly affected by the earnings test of the US social security system which we do not model in detail. Below age 65, however, the earnings test of the US social security system is roughly offset by the benefit recomputation formula that replaces benefits lost (see French, 2005). Table 1 collects the benchmark parameter values.

[5] See http://www.ssa.gov/OACT/ProgData/taxRates.html.

β	σ	ρ	ϕ	μ	τ	θ	λ	f	s
0.980	2	0	0.1	0.576	0.246	0.153	0	0	0.083

Table 1: Parameter values

4 Results

4.1 Labor supply behavior

We first illustrate the labor supply behavior in the model. In figure 1, we display the fraction of individuals supplying a strictly positive amount of labor at different ages. In figure 2, we display the average labor supply of those who provide a strictly positive amount of labor. We show the results with and without an earnings test. The case with an earnings test corresponds to $\lambda = 0.2$ and $f = 0$. Our results are based on simulations of 2000 individuals with randomly determined labor productivity paths. We use the same set of labor productivity paths in all our experiments.[6]

[6] Because of the discrete nature of the labor supply decision at the extensive margin, we solve the optimization problem (given the value function), instead of interpolating the policy functions, at each stage when simulating the lifecycles. This makes simulations somewhat time consuming with a large number of individuals. We checked that 2000 individuals is enough by computing the first set of results in table 3 with 4000 individuals. The results were indeed very close to those presented here.

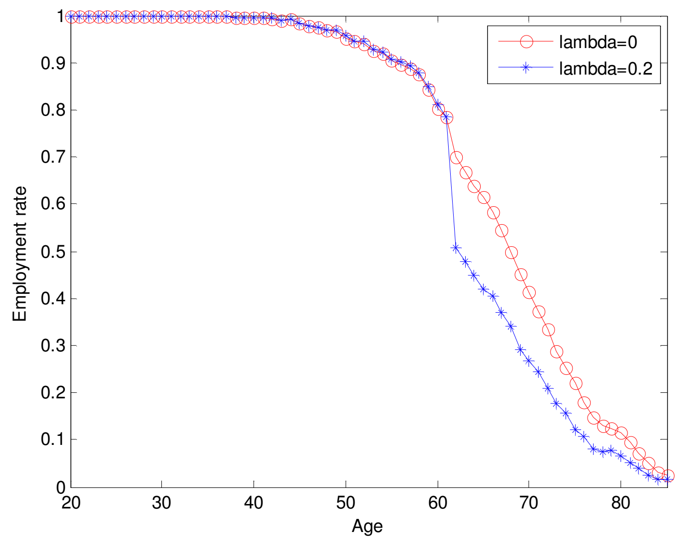

Figure 1. Employment rate with and without earnings testing.

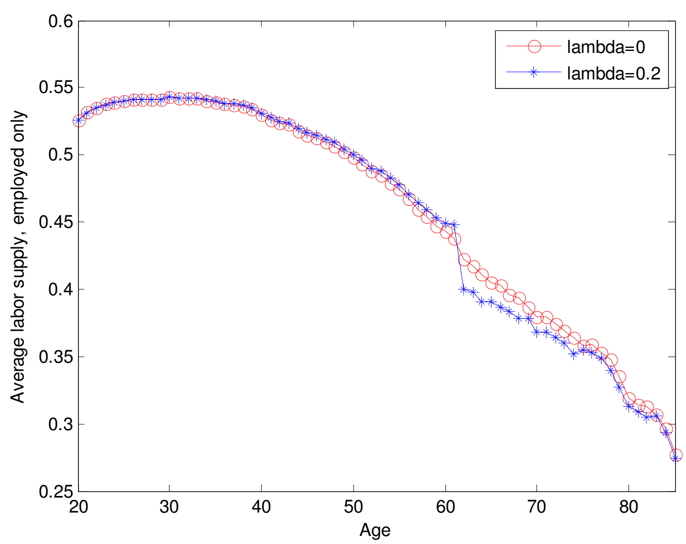

Figure 2. Average labour supply for employed individuals with and without earnings testing.

These profiles are similar to those observed in the real data (as displayed, for instance, in French, 2005). As in the data, the employment rate falls much more rapidly at older ages than average hours worked among those who are employed. This reflects the importance of the extensive margin in the individual labor supply decision.

The earnings test has a large effect on labor supply: the employment rate falls dramatically at the entitlement age. Hence, many of those who work little in the absence of earnings testing, decide not to work at all when benefits are earnings-tested. Note also that even without earnings testing, the participation rate falls somewhat abruptly at the entitlement age. This is because some households are borrowing constrained just before the entitlement age.

The labor supply effects of the earnings test in our model are similar to the results in French (2005) who considers the effects of abolishing earnings testing in the US social security system. Somewhat relatedly, Börsch-Supan (2000) estimates that retirement before the age of 60 could be reduced in Germany by more than a third if the social security system were made actuarially fair.[7] While the German system that Börsch-Supan analyzes imposes an implicit tax on continued activity in the form of less than actuarial adjustment when retirement is postponed, in our analysis the implicit tax arises from explicit earnings testing after the entitlement age.

A key issue here is how changes in the effective tax rate on labor income affect labor supply. The elasticity of labor supply with respect to changes in the net wage rate depends on the distribution of individuals over their savings and labor productivity. Therefore, it is different at different ages. We compute the elasticity of labor supply by experimenting with an anticipated wage increase of 10% for one year at certain ages and comparing average hours worked (across all individuals of a certain age) with and without the wage increase.[8] Table 2 displays the results. The elasticity of labor supply is very high at older ages. Intuitively, at older ages, many individuals are close to being indifferent between participating in the labor market or retiring. Even a small change in the economic incentives may change their decision.

[7] With actuarial adjustment, the net present value of lifetime social security benefits does not depend on retirement behavior. Thus, we define actuarial fairness to be fairness at the margin.

[8] In contrast to our other experiments, this is a partial equilibrium experiment. I.e., prices, taxes and transfers are kept constant. Note that the computed elasticities include a (small) wealth effect.

Age	Labor supply elasticity
30	0.29
50	0.49
60	0.95
65	2.44

Table 2: Estimated labor supply elasticities

4.2 Policy analysis

We analyze the effects of earnings testing by comparing the benchmark economy, which features no earnings testing of old-age benefits, to economies where social security is earnings-tested. In these experiments, we take into account all the general equilibrium effects. Reducing earnings testing affects the interest rate, r, the wage rate, w, the full benefit, s, and the income tax rate, τ, through changes in labor supply and aggregate savings.

Table 3 displays the main aggregate effects, namely the percentage changes in the contribution rate, labor supply, capital stock, interest rate, and the wage rate following the introduction of an earnings test. The results in the table refer to percentage changes compared to the benchmark case. We first set the implicit tax rate, λ, at 0.1, 0.2, and 0.3 without introducing an exemption. We then set λ at 0.2 and introduce an exemption equal to 0.3, 0.2, and 0.1. As shown in table 1, the full benefit is about 0.08.

Setting $\lambda = 0.1$, for instance, allows to increase the full benefit by 3.3%. At the same time, it decreases aggregate effective labor supply by 1.9% and aggregate savings by 1.7%. These changes induce small changes in the interest and wage rates. Setting λ at 0.2 further increases the full benefit, little less than 5% compared to the benchmark case with no earnings test. However, increasing the implicit tax rate further at 0.3 lowers the full benefit compared to the case where the implicit tax rate is set at 0.2. Increasing the implicit tax rate decreases labor supply thereby reducing social security contributions. Reduced earnings at older ages also mean that some individuals may collect higher rather than lower benefits even though the earnings test is made more severe. In short, there is a Laffer curve here: The full benefit can be increased by introducing a moderate earnings test but at some point further increasing the severity of the earnings test lowers the full benefit. The peak of this Laffer curve is between $\lambda = 0.2$ and $\lambda = 0.3$, when there is no exemption.[9]

A social security system with $\lambda = 0.2$ and $f = 0.3$ implies only a slightly higher full benefit than in the benchmark case with no earnings test at all. Keeping the implicit tax rate fixed at 0.2, the earnings test can be made more severe by lowering the exemption. This leads again to an increase in the full benefit and a decrease in labor supply.

λ	f	$\triangle s$	$\triangle L$	$\triangle K$	$\triangle r$	$\triangle w$
0.1	0.0	3.2	−1.9	−1.7	−0.3	0.1
0.2	0.0	4.6	−3.6	−2.9	−1.0	0.3
0.3	0.0	4.2	−5.1	−3.7	−2.1	0.5
0.2	0.3	0.7	−0.7	−0.8	0.3	−0.1
0.2	0.2	1.4	−1.1	−1.3	0.2	−0.0
0.2	0.1	3.1	−2.1	−2.1	−0.0	0.0

Table 3: Aggregate effects of means testing (in percents)

Our main interest lies in the welfare effects. Our welfare measure is the consumption equivalent variation defined as the percentage increase in consumption in all periods and after all possible labor productivity histories needed in the benchmark case to make the expected lifetime welfare as high as in a comparison case. We compute it separately for individuals with a low and a high permanent productivity shock. Let cev_l and cev_h denote, respectively, the consumption equivalent variations for new born individuals with a low and a high permanent shock. In addition, we use cev_u, where u stands for utilitarian, to denote the consumption equivalent variation for new born individuals before they know their permanent productivity shock.

In table 4, we display the changes in the expected lifetime utility following from the introduction of an earnings test. Increasing the implicit tax rate to 0.1, decreases the expected

[9]We checked this by verifying that increasing λ from 0.20 to 0.21 increases the full benefit.

lifetime utility of an individual with a low permanent productivity shock by 0.6% and that of an individual with the high permanent shock by 0.9%. The corresponding loss in the expected welfare before the permanent shock is revealed is 0.8%. The utilitarian welfare loss becomes as large as 2.2% if the implicit tax rate is set at $\lambda = 0.3$.

Decreasing the exemption, while keeping the implicit tax rate at $\lambda = 0.2$, has very similar welfare effects as just increasing the earnings test rate. Therefore, it seems that introducing an exemption does not substantially change the way that earnings testing redistributes life time income across individuals with different permanent income shocks or insures them against temporary shocks.

The results in table 4 show that even a small ($\lambda = 0.1$) earnings test lowers individuals' expected lifetime utility. In other words, its possible risk sharing benefits are not sufficiently large to overcome the efficiency cost of increasingly high effective marginal tax rates on labor. We also searched for an optimal combination of the implicit tax rate and the exemption. We found that having no earnings testing at all maximizes the expected welfare of both the low-ability and the high-ability type.

λ	f	cev_l	cev_h	cev_u
0.1	0.0	-0.6	-0.9	-0.8
0.2	0.0	-1.3	-1.8	-1.5
0.3	0.0	-2.0	-2.6	-2.2
0.2	0.3	-0.3	-0.5	-0.4
0.2	0.2	-0.4	-0.7	-0.5
0.2	0.1	-0.7	-1.1	-0.9

Table 4: Welfare effects of introducing an earnings test

Instead of *expected* lifetime utility, people may be more concerned about the distribution of realized or *ex-post* lifetime utilities. It is possible that even though earnings testing decreases expected life time utility, it nevertheless increases the welfare of those who experience the lowest lifetime utility because of many adverse labor productivity shocks. Figure 3 shows how earnings testing shapes the distribution of realized lifetime utilities. The figure plots individuals' realized lifetime utilities in the benchmark case ($x-$axis) together with the percentage change in welfare that follows from the introduction of an earnings test with $\lambda = 0.2$ and $f = 0$.[10] The individuals face the same labor productivity paths in the two cases.

Except for just two individuals, all individuals would have been better-off, *ex-post*, without earnings testing. The situation is very similar with all the combinations of λ and f considered here. While introducing earnings testing appears to hurt, on average, individuals with relatively high lifetime utility more than those with low lifetime utility, many individuals with very low lifetime utilities are substantially worse off because of earnings testing and very few are better off. In other words, earnings testing appears to be detrimental also in a Rawlsian perspective.

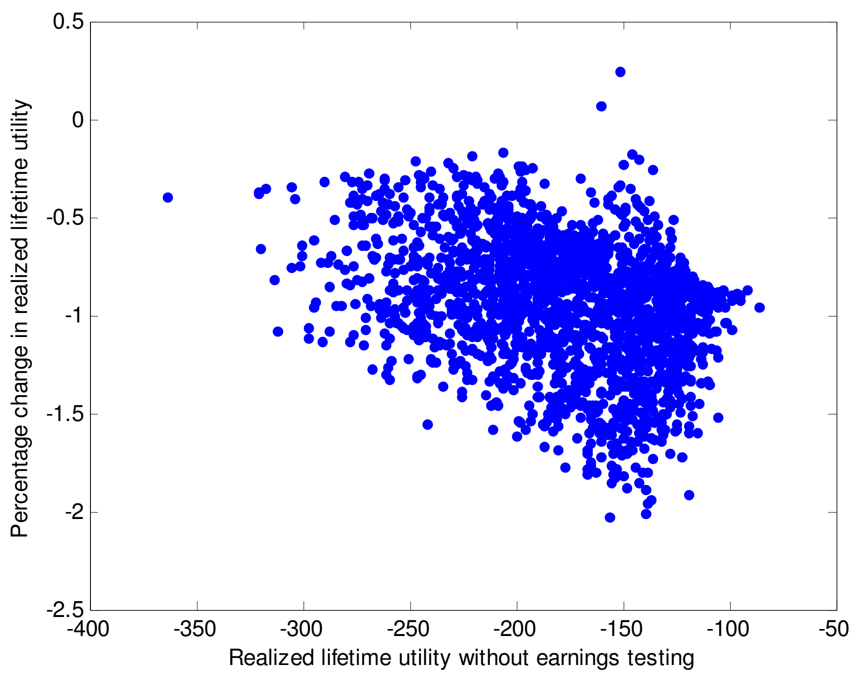

Figure 3. Realized lifetime utility and earnings testing.

[10] We compute here the change in utility, not the equivalent consumption compensation.

As discussed in section 2.3, by fixing the contribution rate, we fix size of the social security system relative to the total wage bill. We have also considered the case where the absolute value of all benefits is fixed. That is, when introducing an earnings test, we adjusted both the contribution rate, θ, and the full benefit, s, so that the sum of all old-age benefits that are paid out is the same as in the initial economy with no earnings test. In that case, following the introduction of an earning test, the full benefit increases more than in the case where the contribution rate is fixed. The welfare results (not shown) were very similar to those reported above. However, for any given earnings test, the welfare losses were slightly larger. This is because the pension system, which causes a steady state efficiency loss, now increases in relation to the wage bill.

4.3 Lifetime earnings and labor supply at old age

Why is it that the earnings test does not improve the insurance properties of social security? For the earnings test to work as insurance against the labor income uncertainty, it should redistribute resources from those who have high lifetime earnings to those with low lifetime earnings. That, in turn, requires that those with high lifetime earnings also tend to earn more after the entitlement age than those with low lifetime earnings.

Figure 4 shows the relation between lifetime earnings and earnings at age 62, which is the age at which individuals start to receive old-age benefits, in the absence of earnings testing. The correlation between lifetime earnings and earnings at age 62 is positive. This is because labor productivity shocks are very persistent. However, the relation is not very systematic: many households with relatively low lifetime earnings have relatively high earnings at age 62, while many individuals with high lifetime earnings earn very little or nothing at age 62. Hence, earnings testing is a relatively inefficient way of targeting higher benefits to those with low lifetime earnings. As a result, it is also an inefficient way of providing insurance against labor income risk.

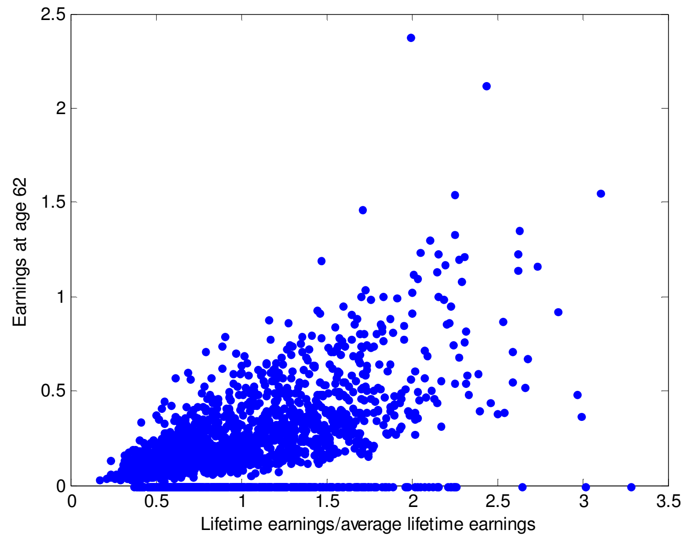

Figure 4. Lifetime earnings vs. earnings at age 62.

It is, of course, easy to find examples where an individual who is poorer in terms of lifetime earnings, chooses to earn more after the entitlement age than a richer individual. That is likely to happen whenever the poorer individual faces a more upward-sloping labor productivity path than the richer one. However, the interaction of individual savings and labor supply under income uncertainty also tends to weaken the correlation between lifetime earnings and earnings at old age. A young individual who is hit by positive productivity shocks saves a large fraction of his income as he expects his future productivity to go down. However, if he continues to be hit by positive shocks, at some point he will start using his savings to consume more leisure. In other words, those who turn out to have high lifetime earnings possibilities are likely to be wealthy at the entitlement age and that makes them retire relatively early.

Figure **5** provides an illustrative example of this mechanism. It depicts the labor supply and savings profiles of two individuals in the case with no earnings test. We compare the cases with and without income uncertainty. In the case with income uncertainty, the individuals have initially the same expectations about their lifetime labor productivity. However, one individual is hit by a small positive shock ($\varepsilon = 0.1$) every period whereas the shock term for the other individual happens to be always zero.[11] The individual who is constantly hit by a positive shock has a much higher average productivity than the other individual, whose productivity follows the average productivity profile. The productivity profile of the high-productivity individual also peaks at a much older age. In the case without income uncertainty, the individuals have the same two labor productivity profiles but they know from the beginning exactly how their labor productivity will evolve over their lifecycle.

Figure 5a shows the labor supply profiles in the two cases. In the deterministic case, the high-productivity individual works more at older ages than the average productivity individual. This is simply because he faces a more upward-sloping productivity profile.[12] In the stochastic case, however, it is the average-productivity individual who works more at old age.

The reason why uncertainty about future labor productivity changes labor supply profiles in this way, is related to the savings behaviour, which is shown in Figure 5b. With uncertainty, the high-productivity individual accumulates large savings already at a young age as he expects his future earnings to be lower. For the same reason, he also works a lot. However, at a certain point the wealth effect associated with large savings starts to dominate his labor supply decision so that he starts to enjoy more leisure than the average-productivity individual.

More generally, the income uncertainty weakens the correlation between lifetime earnings and earnings at the old age through individual savings. This in turn weakens the effectiveness of earnings testing old-age benefits as an insurance or redistribution device.

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[11] This is a partial equilibrium exercise. In particular, the wage and the interest rate are the same with and without uncertainty.

[12] The reason why the two individuals work the same number of hours at young ages, is that they are then both borrowing constrained.

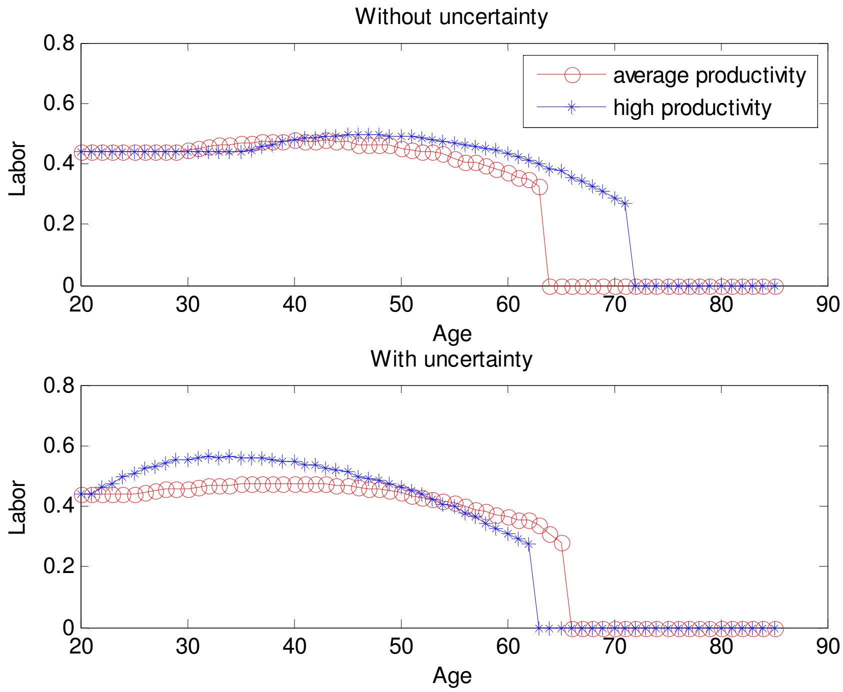

Figure 5a. Examples of labor supply profiles with and without uncertainty about labor productivity.

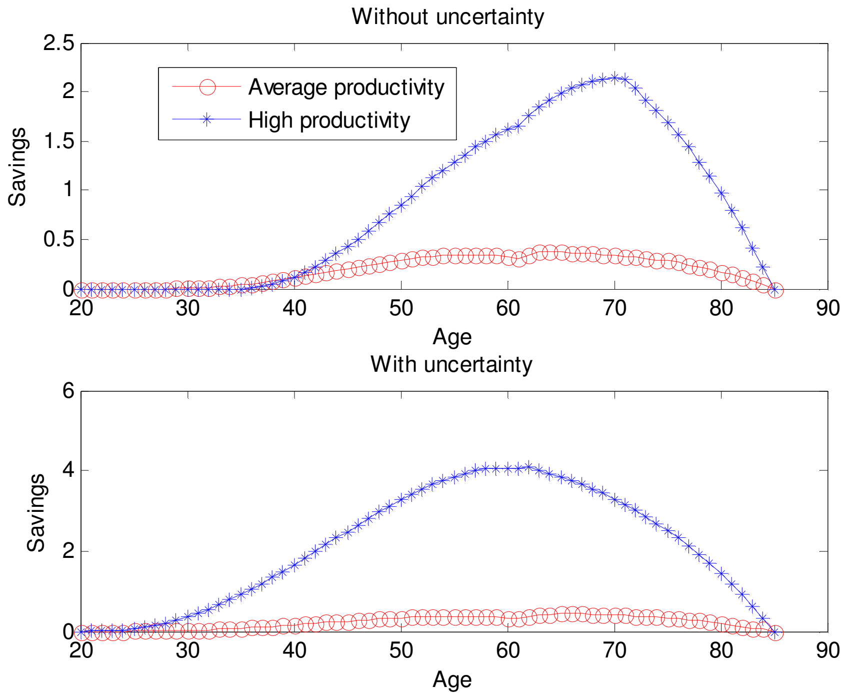

Figure 5b. Examples of savings profiles with and without uncertainty about labor productivity.

5 Sensitivity analysis

5.1 Alternative preference parameters

In this section, we analyze the sensitivity of our welfare results with respect to changes in the preference parameters. We consider changes in the elasticity parameter, ρ, risk-aversion parameter, σ, and the fixed-cost of work, ϕ. In the benchmark calibration we have $\rho = 0$, $\sigma = 2$, and $\phi = 0.1$. We change one of these parameters at a time keeping the other two parameters at their benchmark values. For each specification, we first recalibrate the model so as to match the same targets as in the benchmark case.[13] We then compute the change in the expected lifetime utility that follows from introducing an earnings test with $\lambda = 0.2$

[13] Note that this means that the discount factor and the consumption share parameters are also different across these different calibrations.

and $f = 0$. In addition to the welfare effects, we report the change in the full benefit, s. All these results are reported in percentages.

Table 5 displays the results. For the elasticity parameter, we consider values of 0.5 and -1. The elasticity of substitution between consumption and leisure is given by $\frac{1}{1-\rho}$. Hence, these two values correspond to elasticities of 2 and 0.5, respectively. For the risk aversion parameter we consider values of 3 and 1. For the fixed cost of work, we consider values of 0 and 0.2.

The welfare results are quite sensitive to the elasticity parameter ρ. For $\rho = 0.5$, the ex-ante welfare losses are substantially higher than in the benchmark case and for $\rho = -1$ they are lower. Intuitively, as labor supply becomes increasingly elastic (i.e. ρ is high), distorting the labor supply decision with the earnings-test becomes increasingly expensive in efficiency terms. Related to this, the effect on aggregate labor supply (not reported) is about twice as large with $\rho = 0.5$ than with $\rho = -1$. Despite of this, the change in the full benefit is larger with $\rho = 0.5$ than with $\rho = -1$. The reason appears to be related to the fact that when the labor supply elasticity is high, individuals with a very high productivity spend a much larger fraction of their time endowment working than in the case where the elasticity is low. As a result, when the elasticity is high, there are many individuals that do not receive any old-age benefits at all due to a moderate earnings test. In other words, a wide earnings distribution after the entitlement age works to increase the positive effect that a moderate earnings test has on the full benefit.

The other parameters considered seem to be less important for the welfare results. However, the increase in the full benefit is much higher with a large fixed cost of work than without it.

We also looked at changes in realized lifetime utilities. Except for the case with $\rho = -1$, the share of individuals who enjoy a higher realized lifetime utility due to the introduction of the earnings test is negligible (less than 1%). When $\rho = -1$, about 10% of the individuals are ex-post better off. However, even in this case, most of the individuals with very low realized lifetime utilities are worse off.

	$\triangle s$	$\triangle L$	cev_l	cev_h	cev_u
$\rho = 0.5$	5.7	-5.3	-2.1	-2.9	-2.4
$\rho = -1$	5.8	-1.7	-0.4	-0.5	-0.4
$\sigma = 3$	4.4	-3.4	-1.3	-1.7	-1.5
$\sigma = 1$	5.2	-3.5	-1.3	-1.8	-1.6
$\phi = 0.2$	5.0	-3.6	-1.4	-1.8	-1.6
$\phi = 0.0$	2.5	-2.9	-1.2	-1.5	-1.4

Table 5: Welfare effects of introducing an earnings test with different preference parameters.

5.2 Permanent differences in labor productivity growth rates

In the benchmark specification, the permanent component in the labor productivity process introduces just a level effect. That is, the average productivity profile of low and high-ability types has the same shape. We now consider permanent differences in the expected labor productivity growth rates. We assume that the average labor productivity of high-ability types increases faster at a young age and peaks at a higher age than the average labor productivity of low-ability types. Specifically, we consider the following labor productivity process:

$$w_j = wh_j e^{v^i j + z_j}. \tag{21}$$

Again, we consider two types and assume that $v^1 = -0.01$ (the low-ability type) and $v^2 = 0.01$ (the high-ability type). We assume that the temporary shock follows the same process as in the benchmark case. We recalibrate the endogenously chosen preference parameters to match the same targets as in the benchmark case.

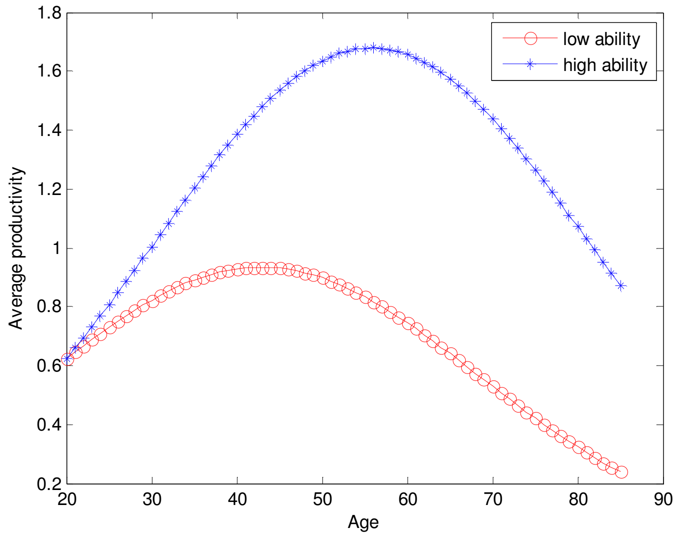

Figure 6. Average productivity profiles with differences in growth rates.

Figure 6 shows the average labor productivity profiles of the two types. These differences in the expected wage growth lead to systematic differences in the labor supply decisions between the two types. Figure 7 shows the average labor supply of the types in the case where the earnings test parameter is set at $\lambda = 0.1$ and there is no exemption, i.e. $f = 0$. At young age, the high-ability types work, on average, slightly less than the low-ability types. At old age, they work much more than the low-ability types. Because of this, we would expect that the earnings test might now distribute income more systematically from the high-ability types to the low-ability types.

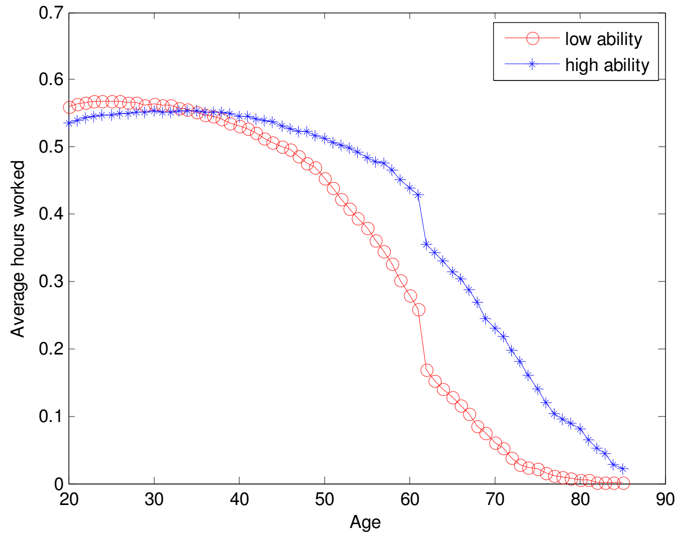

Figure 7. Average labor supply with permanent differences in average productivity growth rates.

Table 6 shows the ex-ante welfare effects of different earnings tests. Compared to the benchmark case (table 4), the welfare losses are bigger for the high-ability type and smaller for the low-ability type. However, introducing an earnings test still always lowers the expected lifetime utility of even the low-ability type. We also considered changes in realized lifetime utilities, and found again that very few individuals would be better off even ex-post following the introduction of an earnings test. These results suggest that reasonable differences in average labor productivity profiles between high- and low-ability types are unlikely to make the earnings test a useful tool for redistributionary purposes.

5.3 The importance of income taxation

We now reconsider the welfare effects of earnings testing assuming that there is no public consumption, i.e. $G = \tau = 0$. This makes it more likely that earnings testing increases individuals' welfare because the marginal tax on labor is initially lower. Again, we recalibrate the model so as to match the other targets. The preference parameters ρ, σ, and ϕ are the same as in the benchmark calibration.

λ	f	cev_l	cev_h	cev_u
0.1	0.0	−0.5	−1.0	−0.8
0.2	0.0	−1.2	−2.1	−1.6
0.3	0.0	−1.9	−2.9	−2.2
0.2	0.3	−0.4	−0.7	−0.5
0.2	0.2	−0.5	−0.9	−0.7
0.2	0.1	−0.7	−1.4	−1.0

Table 6: Welfare effects of introducing an earnings test with permanent differences in productivity growth rates

Table 7 displays the effects of the same earnings tests that were considered in the benchmark case. In contrast to our benchmark results, an earnings test may now increase expected welfare. One somewhat surprising feature of these results is that an increase in the implicit tax rate from 0.2 to 0.3 increases welfare even though it means a slightly lower full benefit. The explanation is related to the general equilibrium effects: Due to its effects on aggregate saving and labor supply, an increasingly high implicit tax rate leads to a higher wage rate, in the case where there is no exemption. This effect increases the steady state lifetime utility.

We also experimented with fixed prices. That is, we assumed that the interest rate and the wage rate remain fixed at their initial steady state levels even as we introduce an earnings test. In that case, all the earnings tests considered here decrease the expected welfare of the high-ability type, whereas moderate earnings tests increase the expected welfare of the low-ability type less than 0.05%. Hence, the welfare effects found here are almost entirely due to the general equilibrium price effects. Of course, the same effects are present in the benchmark case as well. It is just that there the distortionary effects of a higher implicit tax rate dominate.

These results highlight the importance of taking other forms of taxation into account when analyzing the effects of social security. Neglecting other public expenditures not only changes the quantitative results, but may even reverse qualitative conclusions. When individuals decide on their labor supply, they take into account both wage taxes and social security contributions. Neglecting wage taxes results in a severe underestimation of the labor supply distortion that is convex in the overall tax rate.

λ	f	$\triangle s$	$\triangle L$	$\triangle w$	cev_l	cev_h	cev_u
0.1	0	5.97	-1.83	0.21	0.20	0.04	0.14
0.2	0	8.98	-3.46	0.53	0.41	0.13	0.29
0.3	0	8.94	-4.56	0.89	0.58	0.27	0.45
0.2	0.3	1.07	-0.63	-0.01	0.02	-0.04	-0.01
0.2	0.2	2.41	-1.07	0.00	0.06	-0.06	0.00
0.2	0.1	5.47	-1.91	0.08	0.15	-0.07	0.05

Table 7: Welfare effects of introducing an earnings test without income taxation.

6 Conclusions

We have analyzed the welfare effects of earnings testing flat-rate old-age benefits. We used a calibrated overlapping generations model with endogenous labor supply and retirement behavior. We took into account both innate ability differences and temporary shocks arriving during the working life. We found that given a realistic level of general income taxation, flat-rate benefits without earnings testing deliver a higher expected lifetime utility than flat-rate benefits with earnings testing. Flat-rate benefits without earnings testing dominate also from a Rawlsian perspective.

When interpreting our results to real world policy debates, one must keep in mind at least two things. First, we have considered only labor income shocks and not, for instance, disability shocks. Clearly, earnings testing old-age benefits may help in targeting benefits to the disabled if these risks cannot be addressed directly with other insurance schemes. Second, social security systems can be divided into flat-rate, or 'Beveridgean', and earnings-related, or 'Bismarckian', systems.[14] Our model applies to Beveridgean systems, and a corresponding

[14]Beveridgean tradition is dominant in Anglo-Saxon countries, including the United States and the United Kingdom, as well as in Denmark and the Netherlands. Most of Continental Europe follows a competing Bismarckian tradition in which benefits are more closely related to past earnings (Disney, 2004).

analysis would be needed to draw policy conclusions for Bismarckian systems.

References

[1] Börsch-Supan, A. (2000). Incentive Effects of Social Security on Labor Force Participation. *Journal of Public Economics* 78 (1-2): 25-49.

[2] Conesa, J.C. and D. Krueger. (1999). Social Security Reform with Heterogenous Households. *Review of Economic Dynamics* 2 (4): 757-795.

[3] Cremer, H., J.-M. Lozachmeur, and P. Pestieau. (2004). Social Security, Retirement Age and Optimal Income Taxation. *Journal of Public Economics* 88 (11): 2259-2281.

[4] Diamond, P.A., and J.A. Mirrlees. (1978). A Model of Social Insurance with Variable Retirement. *Journal of Public Economics* 10 (3): 295-336.

[5] Diamond, P.A., and J.A. Mirrlees. (1986). Payroll-Tax Financed Social Insurance with Variable Retirement. *Scandinavian Journal of Economics* 88 (1): 25-50.

[6] Disney, R. (2004). Are Contributions to Public social security Programmes a Tax on Employment. *Economic Policy* 39: 267-311.

[7] Fehr, H. and C. Habermann. (2008). Risk Sharing and Efficiency Implications of Progressive Social Security Arrangements. Scandinavian Journal of Economics 110 (2): 419-443.

[8] Floden, M. and J. Lindé. (2001). Idiosyncratic Risk in the United States and Sweden: Is There a Role for Goverment Insurance? *Review of Economic Dynamics* 4 (2): 406-437.

[9] French, E. (2005). The Effects of Health, Wealth, and Wages on Labour Supply and Retirement Behaviour. *Review of Economic Studies* 72 (2): 395-427.

[10] Friedman, M. (1953). Choice, Chance and the Personal Distribution of Income. *Journal of Political Economy* 61 (4): 277-290.

[11] Golosov, M. and A. Tsyvinski. (2006). Designing Optimal Disability Insurance: A Case for Asset Testing. *Journal of Political Economy* 114 (2): 257-269.

[12] Gomes, F. and A. Michaelides. (2003). Aggregate Implications of Defined Benefit and Defined Contribution Systems. Working paper, Center for Retirement Research at Boston College. <http://escholarship.bc.edu/cgi/viewcontent.cgi?article=1071&context=retirement_papers>

[13] Gruber, J. and D.A. Wise. (1999). Social Security and Retirement around the World. University of Chicago Press, Chicago IL.

[14] Gustman, A. and T. Steinmeier. (1986). A Structural Retirement Model. *Econometrica* 54(3): 555-584.

[15] Harsanyi, J.C. (1953). Cardinal Utility in Welfare Economics and in the Theory of Risk-taking. *Journal of Political Economy* 61 (5): 434-435.

[16] İmrohoroğlu, A., S. İmrohoroğlu, and D.H. Joines. (1995). A Life Cycle Analysis of Social Security. *Economic Theory* 6 (1): 83-114.

[17] Rawls, J. (1971). A Theory of Justice. Harvard University Press, Cambridge, MA.

[18] Rust, J. and C. Phelan. (1997). How Social Security and Medicare Affect Retirement Behavior in a World of Incomplete Markets. *Econometrica* 65 (4): 781-831.

[19] Sefton, J., J. van de Ven, M. Weale. (2008). Means Testing Retirement Benefits: Fostering Equity or Discouraging Savings? *Economic Journal* 118 (528): 556-590.

[20] Social Security Administration. (2005). http://www.ssa.gov/policy/docs/progdesc/ssptw/2002-2003/americas/united_states.html

[21] Stock, J. and D. Wise. (1990). An Option Value Model of Retirement. *Econometrica* 58 (5): 1151-1180.

A Computational issues

In this appendix we explain how we solve the individual's problem.

The individual's problem is solved recursively with value function iteration. We discretize the two continuous state variables, k_j and z_j. This forms a two-dimensional grid. We use interpolation to determine the value function between gridpoints. For each gridpoint, we need to solve for the optimal savings and labor supply.

The savings problem is dynamic and is solved with a numerical optimization algorithm. However, for a given savings decision (given k_{j+1}), the labor supply problem is static. This implies that we can make use of analytical derivations.

Specifically, for a given k_{j+1}, and assuming that the individual participates in the labor market ($l > 0$), the labor supply problem reads as:

$$\begin{aligned} & \max_l \frac{[(\mu * c^\rho + (1-\mu)(1-l-\phi)^\rho)^{\frac{1}{\rho}}]^{1-\sigma}}{1-\sigma} \\ & s.t. \\ c + k_{j+1} &= Rk_j + (1-\theta-\tau)w_j l + B(w_j l, j). \end{aligned}$$

where $R = [1 + r(1-\tau)]$.

For ages $j \geq 62$, we have the following benefit rule (for $j < 62$ the problem is much simpler):

$$B(wl, j) = \max[s - \lambda(1-\tau-\theta)\max[w_j l - f, 0], 0].$$

In order to solve this problem, we consider three cases:

$$\begin{aligned} 1)\ s - \lambda(1-\tau-\theta)\max[w_j l - f, 0] &\leq 0 \\ 2)\ s - \lambda(1-\tau-\theta)\max[w_j l - f, 0] &> 0 \text{ and } w_j l - f \geq 0 \\ 3)\ s - \lambda(1-\tau-\theta)\max[w_j l - f, 0] &> 0 \text{ and } w_j l - f < 0. \end{aligned}$$

In each case, we can write the budget constraint as $c_i = A_i + B_i l_i$, where $i \in \{1, 2, 3\}$ and

$$\begin{aligned}
A_1 &= Rk_j - k_{j+1} \\
B_1 &= (1-\theta-\tau)w_j \\
A_2 &= Rk_j - k_{j+1} + s + \lambda(1-\theta-\tau)f \\
B_2 &= (1-\theta-\tau)w_j - \lambda(1-\theta-\tau)w_j \\
A_3 &= Rk_j - k_{j+1} + s \\
B_3 &= (1-\theta-\tau)w_j.
\end{aligned}$$

Hence, the problem can be written as

$$\max_{l_i} \mu(A_i + B_i l)^\rho + (1-\mu)(1 - l_i - \phi)^\rho.$$

The first-order condition reads as:

$$\begin{aligned}
\mu B_i (A_i + B_i l_i)^{\rho-1} &= (1-\mu)(1 - l_i - \phi)^{\rho-1} \\
(\mu B_i)^{\frac{1}{\rho-1}} (A_i + B_i l_i) &= (1-\mu)^{\frac{1}{\rho-1}} (1 - l_i - \phi).
\end{aligned}$$

We write this as

$$\begin{aligned}
D_i(A_i + B_i l_i) &= F_i(1 - l_i - \phi) \text{ where} \\
D_i &= (\mu B_i)^{\frac{1}{\rho-1}} \text{ and} \\
F_i &= (1-\mu)^{\frac{1}{\rho-1}}.
\end{aligned}$$

The interior solutions is

$$l_i = \frac{F_i - F_i\phi - A_i D_i}{D_i B_i + F_i}.$$

Hence, we have three unconstrained solutions, l_1, l_2, and l_3. We first check for possible corner solutions:

$$\begin{aligned}
l_1 &\geq \frac{s + \lambda(1-\theta-\tau)f}{\lambda(1-\theta-\tau)w} \\
\frac{f}{w} &\leq l_2 \leq \frac{s + \lambda(1-\theta-\tau)f}{\lambda(1-\theta-\tau)w} \\
0 &\leq l_3 \leq \frac{f}{w}
\end{aligned}$$

As a result, we have three consumption-labor pairs and we simply choose the one that yields the highest utility. We then compare that outcome to the one that corresponds to not participating in the labor market (in which case there is no fixed cost ϕ in the utility function).