Macroeconomics 1 - Problem set 4

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Due 10.10.2018. (Hand in your answers in the exercise class or send them to lauro.carnicelli (at) helsinki.fi before the class. NOTE: this last exercise class starts at 10am.)

1. Consider household savings decisions with habit formation. Habit formation refers to the idea that current utility may depend not just on current consumption but also on past consumption. It has been used to explain the so called equity premium puzzle as well as aggregate consumption dynamics over the business cycle.

Specifically, assume that the household's periodic utility is given by

$$u(c_t, h_t) = \frac{\{c_t/h_t^{\gamma}\}^{1-\rho}}{1-\rho} \tag{1}$$

where c_t denotes consumption and h_t habit level. The parameter $\gamma \in [0,1]$ captures the importance of habits. Habit level evolves as

$$h_{t+1} = (1-\lambda)h_t + \lambda c_t \tag{2}$$

where $\lambda \in [0,1]$.

The household lives for periods $t = 1, 2, ..., T$. Each period t, it chooses consumption c_t and savings a_{t+1} to maximize expected remaining lifetime utility

$$E_t \sum_{t=1}^{T} \beta^{t-1} u(c_t, h_t) \tag{3}$$

subject to (2) and

$$\begin{aligned} c_t + a_{t+1} &= (1+r)a_t + \epsilon w_t && (4)\\ & a_1, h_1 \ \textit{given} && (5)\\ a_{T+1} &\geq 0 && (6) \end{aligned}$$

where a_t denotes a financial asset, r interest rate, and w_t age-dependent wage income, and ϵ a discrete $i.i.d$ income shock (the probability distribution for next period shocks is independ of the current shock).

i) Write the household problem recursively. (1p.)

ii) Derive the household's first-order condition. (You need not try to eliminate derivatives of the value function.) (1p.)

2. Consider the life cycling savings problem with idiosyncratic income uncertainty in two different economies. In economy 1, the problem of a household of age j is the following:

$$V_j(a_j, s_j) = \max_{c_j}\{\frac{c_j^{1-\sigma}}{1-\sigma} + \beta S_j E_j V_{j+1}(a_{j+1}, s_{j+1})\} \tag{7}$$

subject to

$$\begin{aligned} a_{j+1} &= (1+r)a_j + (1-\tau)s_j w_j + b_j - c_j & (8)\\ a_{j+1} &\geq 0 & (9)\end{aligned}$$

where a denotes financial savings, $S_j \leq 1$ survival probability, b_j a flat rate pension benefit (zero before retirement age), and s the income shock (first-order Markov).

Note that those who die may leave some assets. Those assets are often referred to as "accidental bequests". In this economy, there is a malevolent government that confiscates all accidental bequests and uses them for something that is of no value to the households.

In economy 2, there is a competitive insurance company that sells "annuities" (denoted here also by a). The company invests the money it receives by selling the annuities to the capital market earning an interest rate r. The annuity contract specifies that a household of age j that invests an amount a_{j+1} to these insurance products receives $\frac{(1+r)a_{j+1}}{S_j}$ next period in case she survives. If the individual does not survive, the insurance company keeps the money.

Assuming that the household saves with annuities only, its problem can be written as follows:

$$\widetilde{V}_j(a_j, s_j) = \max_{c_j}\{\frac{c_j^{1-\sigma}}{1-\sigma} + \beta S_j E_j \widetilde{V}_{j+1}(a_{j+1}, s_{j+1})\} \tag{10}$$

subject to

$$\begin{aligned} a_{j+1} &= \frac{(1+r)a_j}{S_{j-1}} + (1-\tau)s_j w_j + b_j - c_j & (11)\\ a_{j+1} &\geq 0 & (12)\end{aligned}$$

i) Show that the household prefers (at least weakly) annuities to the standard asset of economy 1. (1p.)

ii) Assume that there is a continuum (or a very large number) of households buying annuities. Show that the insurance company makes zero profits. (1p.)

iii) Take the calibration corresponding to $MaxAge = 75$ in *lifecycle.m* (with survival probabilities defined in *wages_survivalprobs.xls*). Modify the programs so that you are able to solve the household problem also in economy 2 above. (If you need to divide by S_0, just assume $S_0 = 1$.) Solve the household problem in economy 1 and economy 2. Simulate 1000 life cycles in both economies. Plot the average consumption and savings profiles in the two economies. Hand in the graphs and relevant parts of the code. (2p.)