Macroeconomics 1 - Problem set 2

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Due 19.9.2018. (Hand in your answers in the exercise class or send them to lauro.carnicelli (at) helsinki.fi before the class.)

1. Consider a version of the neoclassical growth model where the social planner maximizes

$$\sum_{t=1}^{\infty} \beta^{t-1}[\gamma \log c_t + (1-\gamma)\log(1-n_t)] \tag{1}$$

subject to

$$\begin{aligned} k_{t+1} + c_t &= Ak_t^{\alpha} n_t^{1-\alpha} + (1-\delta)k_t \ for\ t = 1, 2, ... && (2)\\ k_1 &> 0\ given && (3) \end{aligned}$$

a) Derive the equations that define (and could be used to solve for) the steady state allocation. (1p.)

b) Set $\delta = 1$ and assume A is defined so that steady state output is equal to one. Solve (analytically) for steady state k and c as functions of model parameters. (1p.)

2. Household chooses consumption c for periods 1 and 2, first period labor supply n and first period savings s to maximize $u(c_1, 1-n)+\beta u(c_2, 1)$. It faces the following budget constraints: $c_1+s = wn+y$, $c_2 = (1+r)s$, where r is the interest rate, w the wage rate, and y initial wealth.

a) Derive a set of equations that can be solved to determine the solution to the household problem (assuming you are given the utility function and the parameter values w, y, r and β). For now, you may assume the parameter values are such that the optimal labour supply is strictly positive. (1p.)

b) Assume $u(c, 1-n) = \frac{(c^{\gamma}(1-n)^{1-\gamma})^{1-\sigma}}{1-\sigma}$. Write down a Matlab program to solve the household problem numerically (you may want to see *basicgrowthmodel.m* for inspiration). Hand in the code. (1p.)

c) Set $w = 1, y = r = \beta = \gamma = 0.5$ and $\sigma = 2$. Compute and report c_1, c_2 and n. Also consider how the labour supply changes if you increase either w or y. Explain the results. (1p.)

d) Consider now the possibility that the household might want to choose $n < 0$. Derive the optimality conditions that take into account the constraint $n \geq 0$. Also, try to extend your

answer to part b) accordingly and hand in the code. (1p.)

3. Modify *basicgrowthmodel.m* to allow you to easily calibrate the discount factor and the consumption share parameter in the utility function so that the steady state capital-to-output ratio $k/f(k,n)$ equals 3 and and labour supply $n = 0.333$. (Hint: modify function *ss_equs* so that you solve for the discount factor and the consumption share together with the steady state allocation.) Hand in the relevant parts of the code. (2p.)