Macroeconomics 1 - Problem set 1

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Due 12.9.2018. (Hand in your answers in the exercise class or send them to lauro.carnicelli (at) helsinki.fi before the class.)

1. Show that a Cobb-Douglas production function of the form $K^{\alpha}(AL)^{1-\alpha}$ satisfies the "neoclassical assumptions" listed in section 2.2 of lecture notes 1. (As usual, the factors of production are K and L.) (2p.)

2. Consider a Cobb-Douglas production function of the form $(AK)^{\alpha}(L)^{1-\alpha}$. Show that it can be written as $(K)^{\alpha}(\tilde{A}L)^{1-\alpha}$, where $\tilde{A}$ is a function of A and independent of K and L. (2p.)

3. Modify *Solow.m* or *Solow.jl* so that it allows you to consider a CES production function with capital augmenting technological change of the form $(\alpha(A_tK_t)^{\rho} + (1-\alpha)L_t^{\rho})^{1/\rho}$. Here the elasticity of substitution is given by $\frac{1}{1-\rho}$. Set $A_0 = 1$, $L_0 = 1$, $K_0 = 1$, $g = 0.02$, $n = 0.01$, $\delta = 0.1$, $s = 0.25$, $\alpha = 0.333$ and $\rho = 0.5$. Simulate the economy for 200 periods and plot the evolution of the capital-to-output ratio K/Y. Also, for comparison, plot the capital-to-output ratio assuming a Cobb-Douglas production function of the form $(A_tK_t)^{\alpha}(L_t)^{1-\alpha}$. (2p.)

4. Use the program you wrote for problem 3 to study the implications of negative population growth. Set $g = 0$ (no exogenous productivity growth) and $n < -\delta$. Assume the same CES production function as above and consider different values for ρ, e.g. $\rho = 0.5$ and $\rho = -0.5$. Consider also the Cobb-Douglas function. Simulate the economy for at least 100 periods and plot output per capita Y/L. Interpret the results briefly. (2p.)

5. Consider land (or, more generally, a fixed factor of production) in the Solow growth model. Specifically, let the production function be $A_tK_t^{\alpha}Z^{\beta}(L_t)^{1-\alpha-\beta}$, where Z (land) is constant over time. (Again, $A_{t+1} = (1+g)A_t$.)

a) Assume that in addition to labor and capital, households also own land and rent it to a representative firm. Rewrite the firm problem (see section 2.3 in lecture notes 1) accordingly and derive an expression for the rental rate of land. (2p.)

b) Extend *Solow.m* or *Solow.jl* so that it can be used to simulate this version of the Solow model and also to compute the factor prices. Assume $g > 0$ and $n > 0$ and illustrate - with numerical simulations - how the presence of land influences the dynamics of the model and how the rental rate of land evolves over time. Hand in the code as well. (2p.)